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FIDUCEO workshop Lisbon 17-19th April 2018

Uncertainty Concepts 1:

Basic Principles & Vocabulary

ERROR

is **NOT**

the **same** *as*

Uncertainty

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Measurement, Error & Uncertainty
How a programmer would simulate it

Modelling a measurement process – what is the result of the measurement?

- Define a true value → In the real world you never know this!
- Define a probability distribution → The standard deviation is the uncertainty.
- Get a random value from the distribution → This is the error. In the real world you never know this.
- Add the value to the true value → This simulates the measured value.

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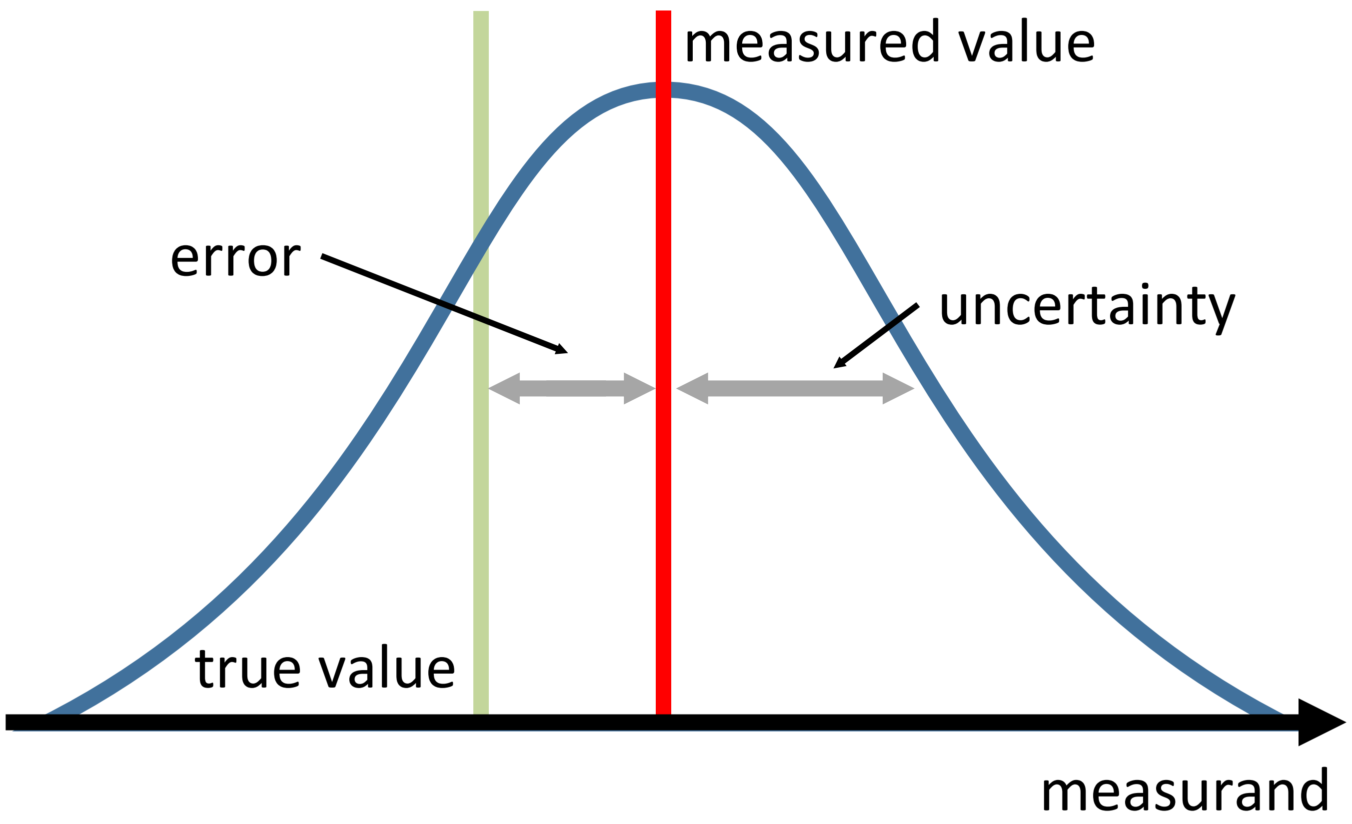
Measurement, Error & Uncertainty



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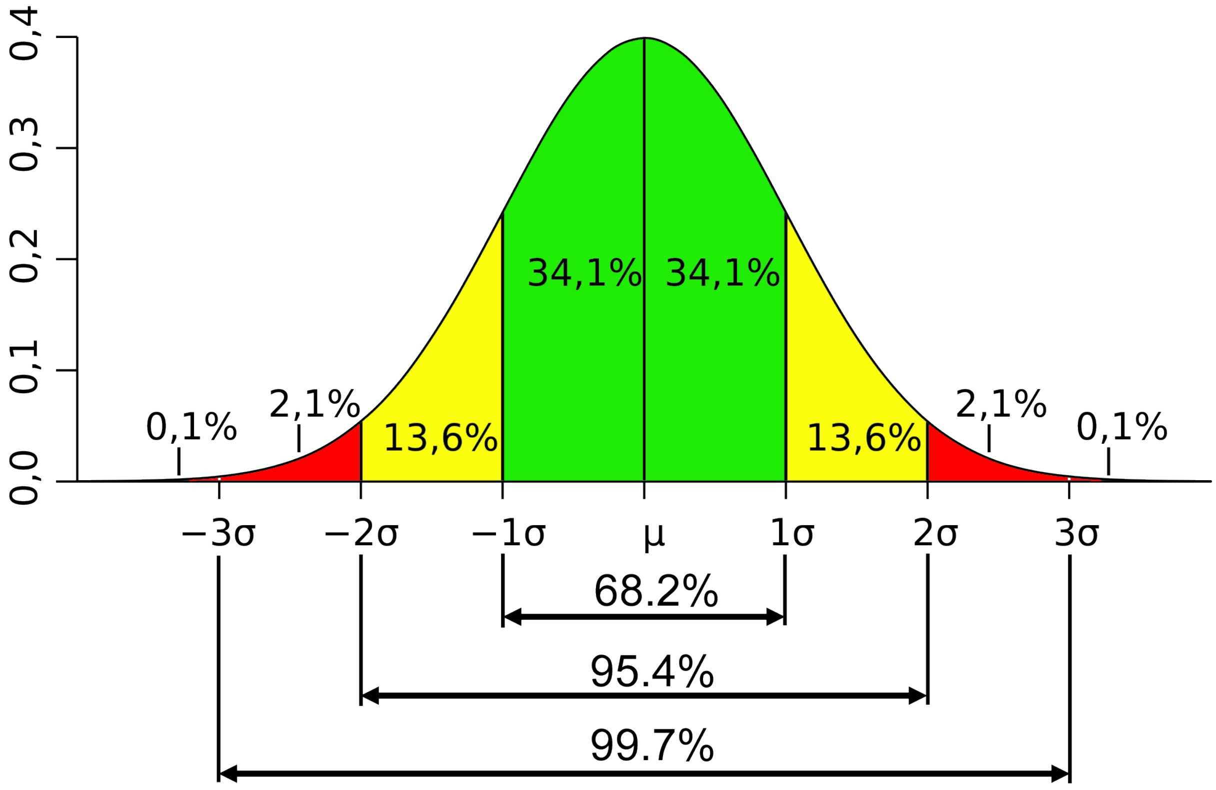
I made a measurement of the measurand 'radiance' to get a measured value of $0.3\ \mathrm{W\ m^{-2}\ sr^{-1}\ nm^{-1}}$ with an associated standard uncertainty of 1 %.

Standard and Expanded Uncertainties



Standard uncertainty:

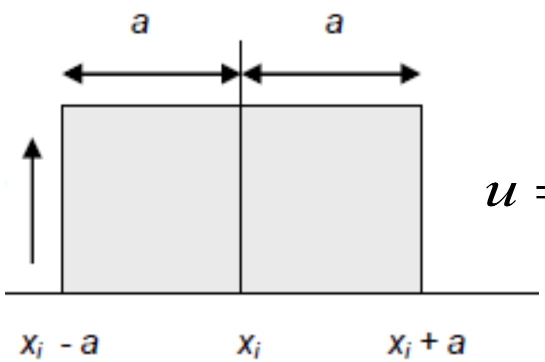
Standard deviation

Expanded uncertainty:

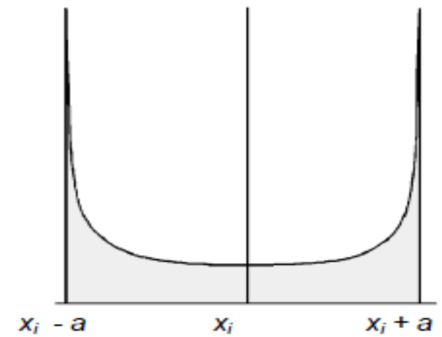
Confidence level with a stated coverage factor

For normal distributions, 95 % confidence level approx. $k = 2$

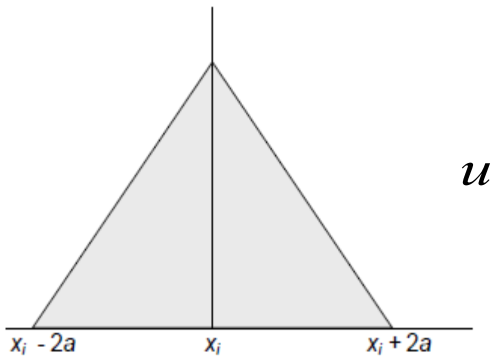
Standard Uncertainties Other Distributions



$$u = a/\sqrt{3}$$

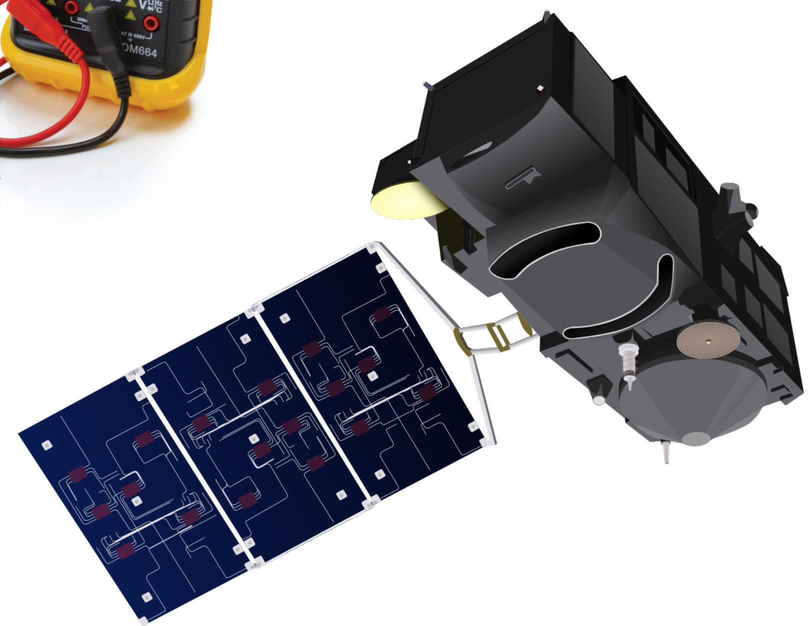


$$u = a/\sqrt{2}$$

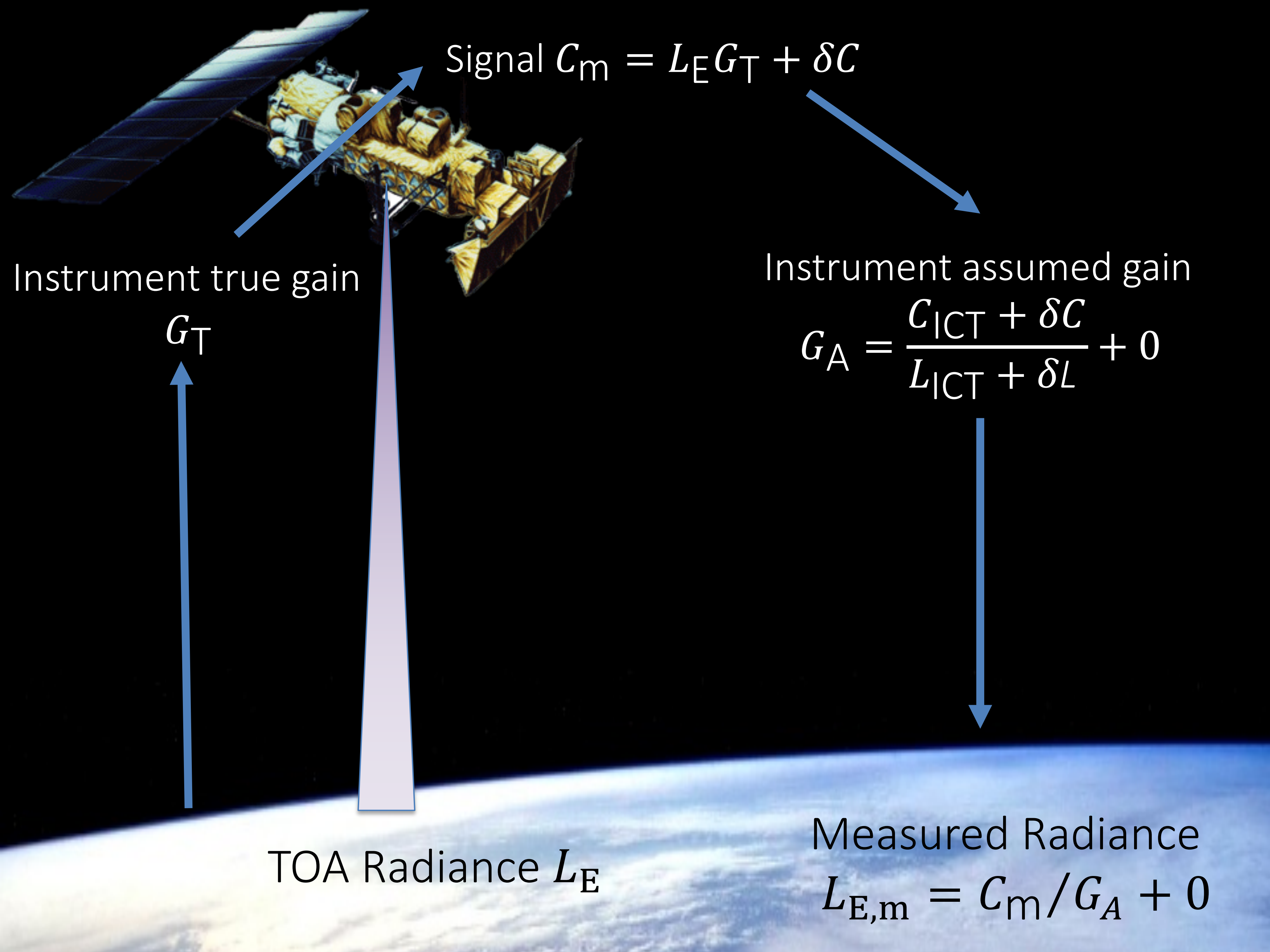


$$u = a/\sqrt{6}$$

These can be looked up!

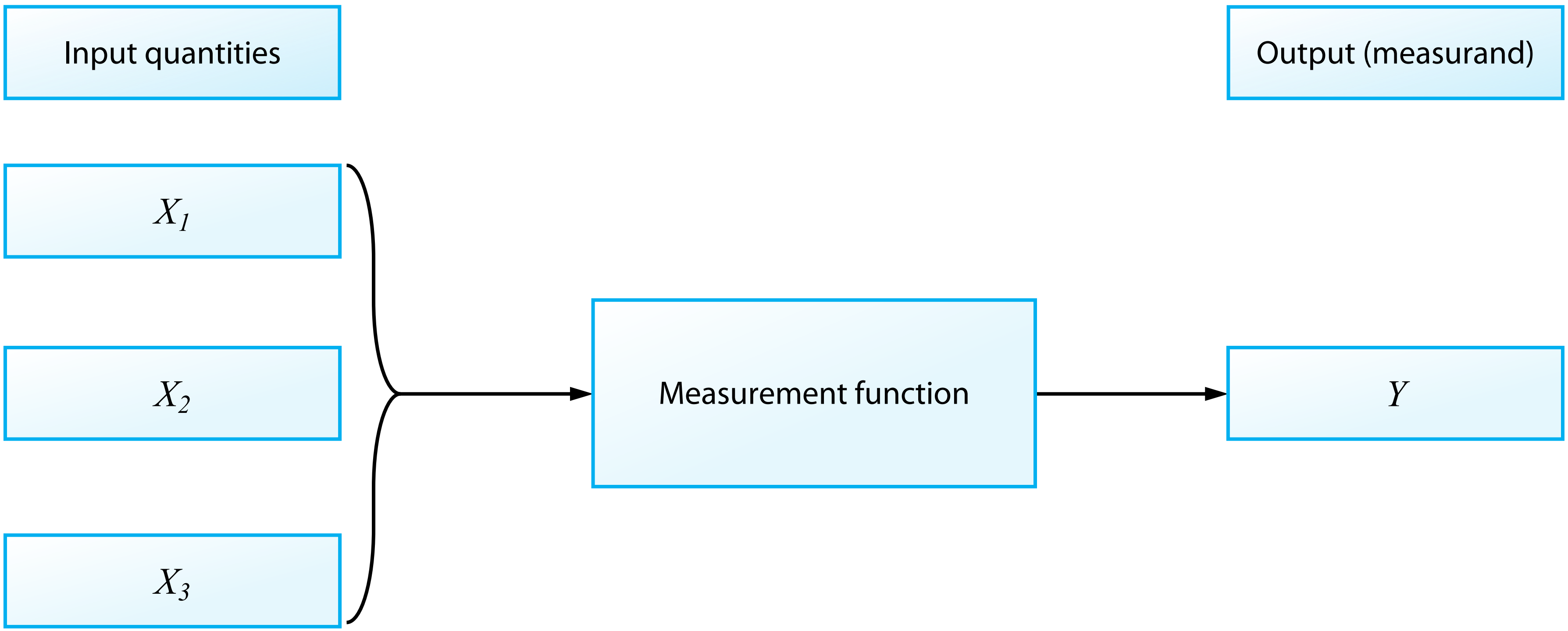


Measurements are usually indirect!

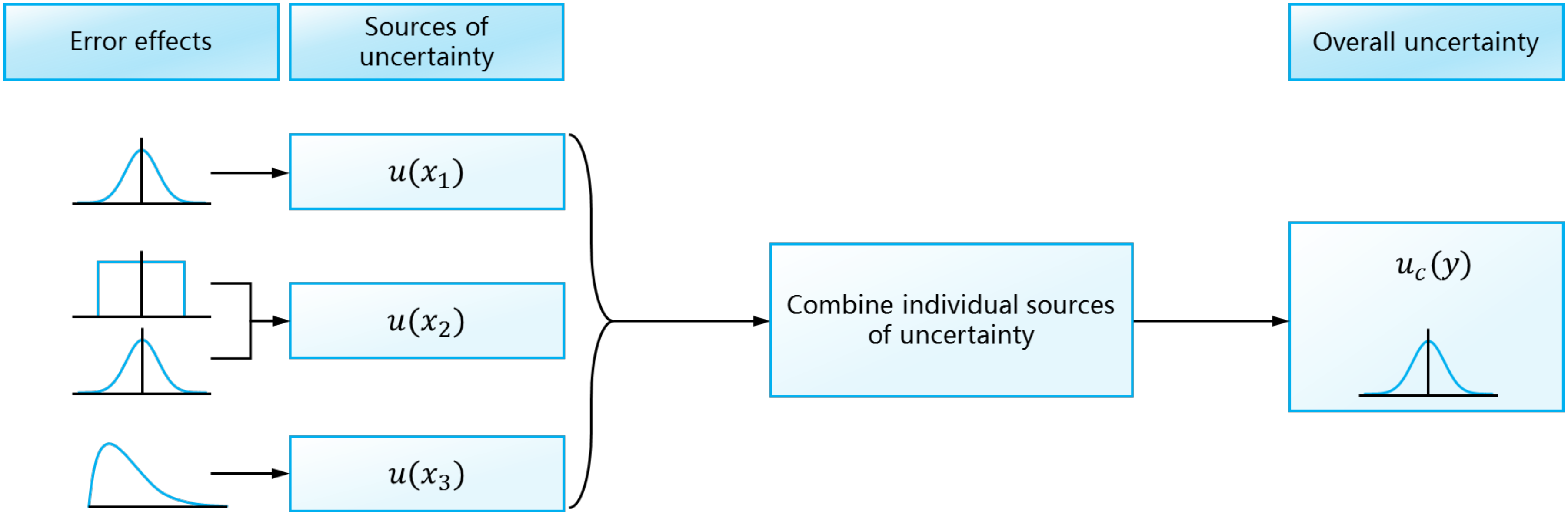


Signal $C_{\mathrm{m}} = L_{\mathrm{E}} G_{\mathrm{T}} + \delta C$
Instrument true gain
G_{T}
Instrument assumed gain
$G_{\mathrm{A}} = \frac{C_{\mathrm{ICT}} + \delta C}{L_{\mathrm{ICT}} + \delta L} + 0$
TOA Radiance L_{E}
Measured Radiance
$L_{\mathrm{E,m}} = C_{\mathrm{m}} / G_{A} + 0$

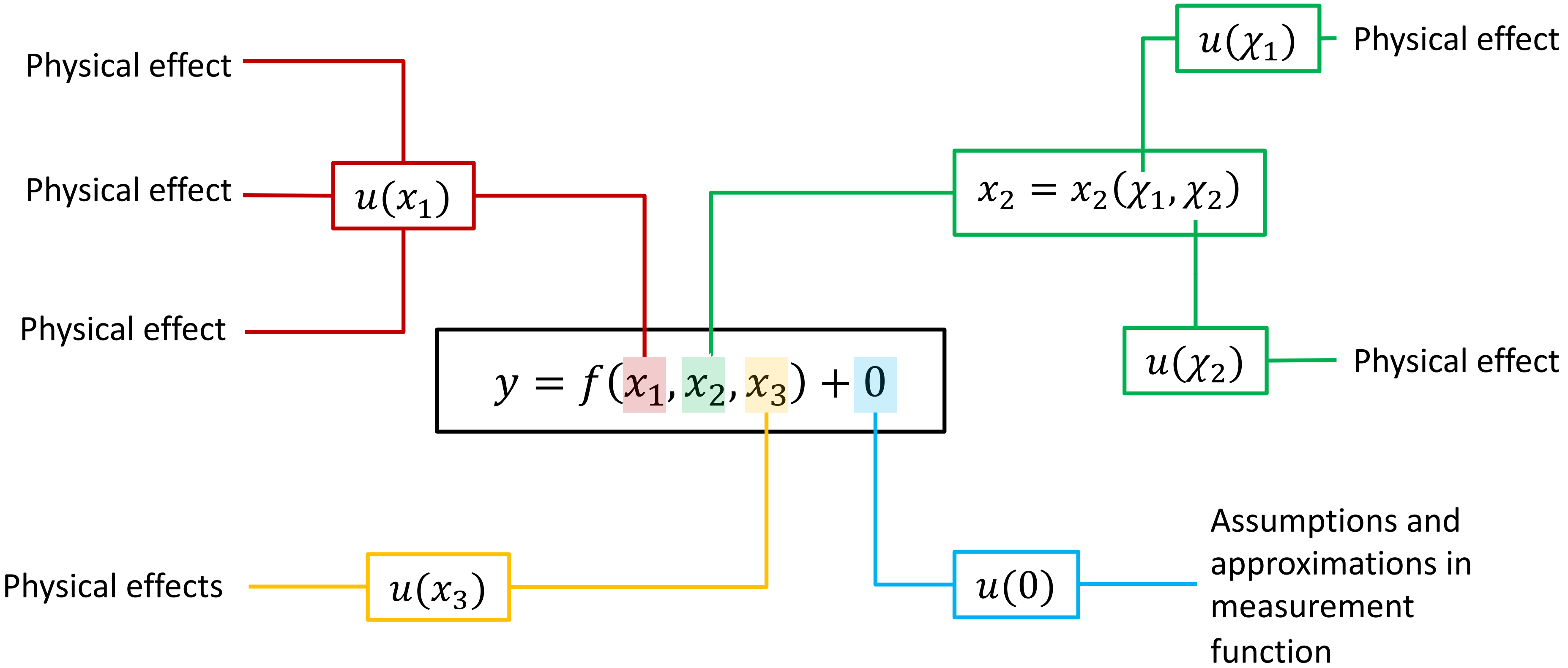
Measurement Functions



Combining Uncertainties



Origins of Errors and Uncertainties

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GUM 1995 with minor corrections

Evaluation of measurement data — Guide to the expression of uncertainty in measurement

Évaluation des données de mesure — Guide pour l'expression de l'incertitude de mesure

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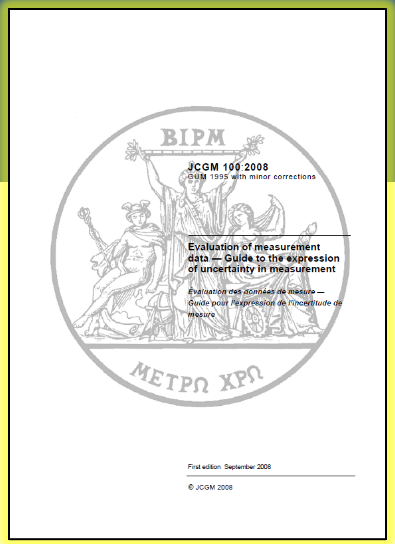
The GUM

https://www.bipm.org/en/publications/guides/

Combining Uncertainties the GUM ways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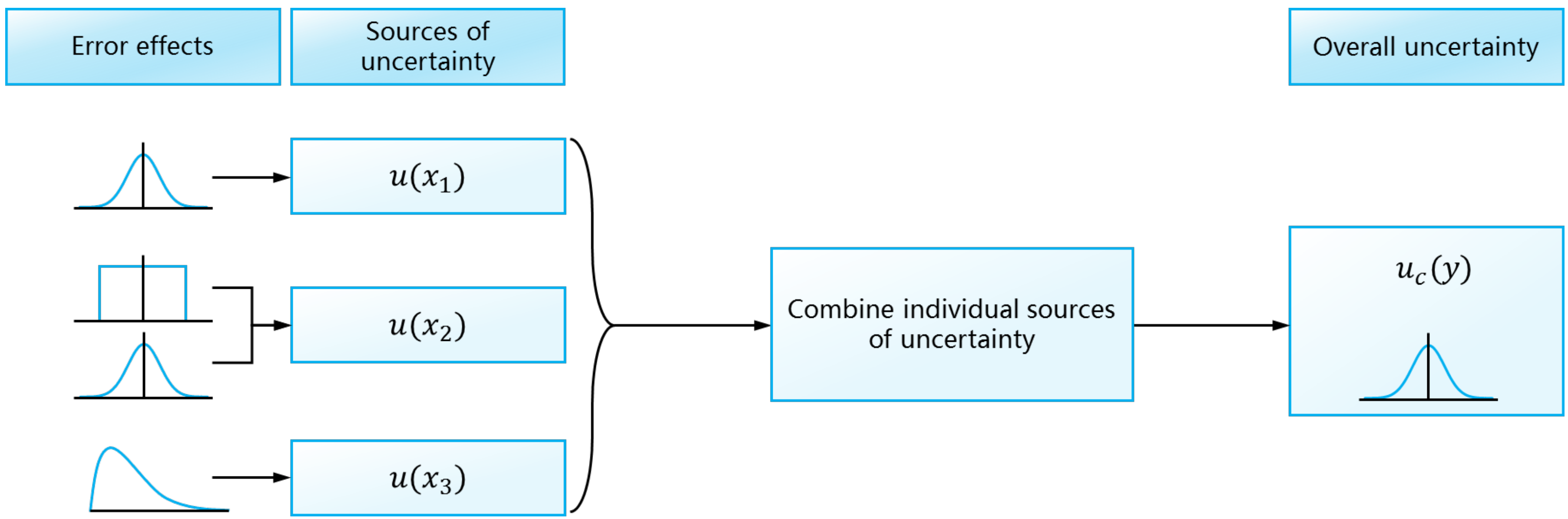
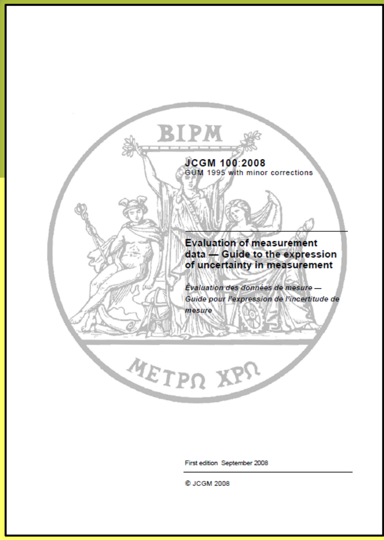
1. Monte Carlo (GUM Supplement 1)
   Approximate by sampling the distribution
2. The Law of Propagation of Uncertainties
   Approximate by locally-linearised model



The Law of Propagation of Uncertainties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$$u_c^2(y) = \sum_{i=1}^{n}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}u(x_i,x_j)$$

Adding in quadrature

Bit that deals with **error correlation**

Law of the Propagation of Uncertainties

If effects uncorrelated,

$$u_c^2(y) = \sum_{i=1}^{n} c_i^2 \, u^2(x_i)$$

- $u_c(y)$ - Combined uncertainty of measurand

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Law of the Propagation of Uncertainties

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- $u_c(y)$ - Combined uncertainty of measurand
- $u(x_i)$ - Uncertainty of each input quantity
- c_i - Sensitivity of the measurand to the input quantity

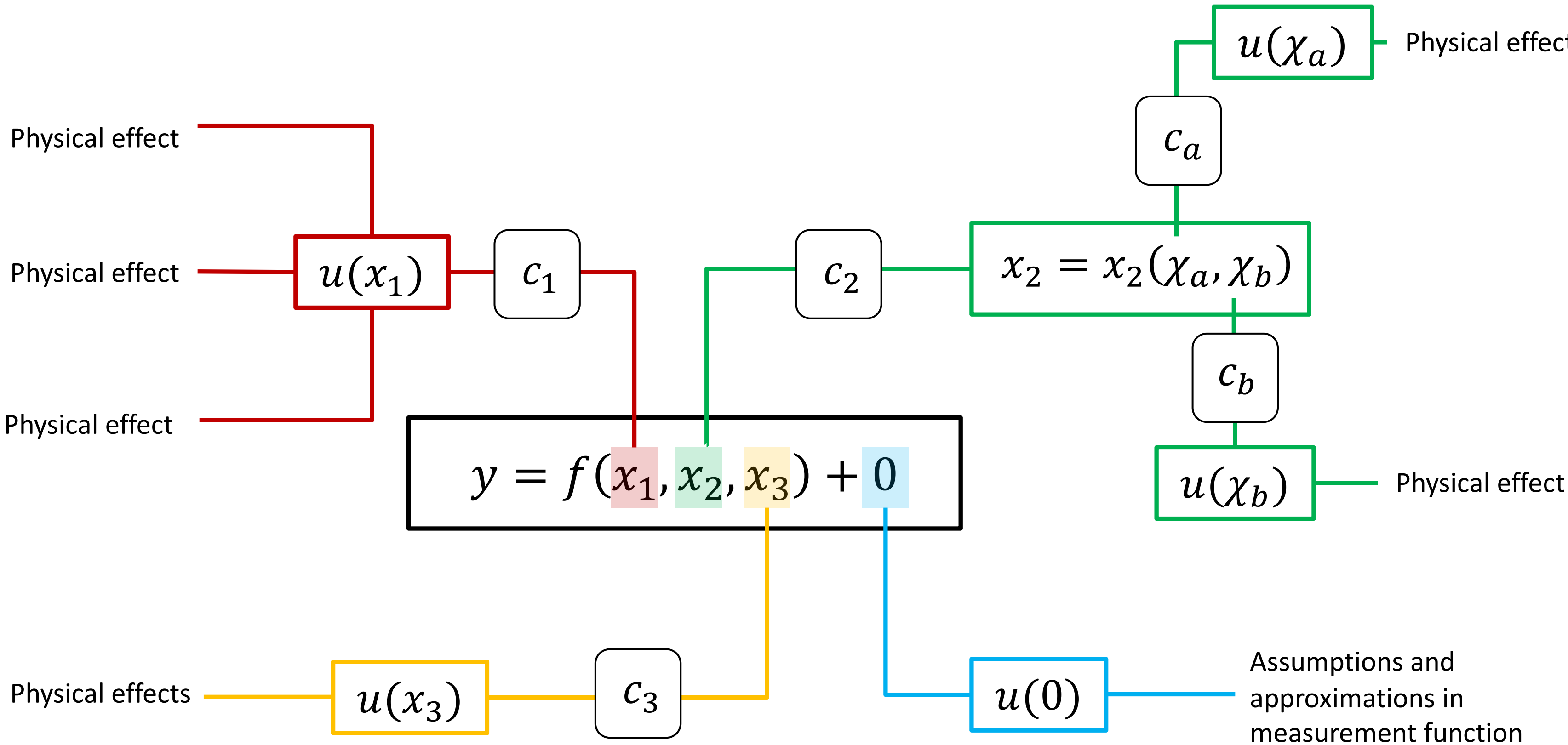
Sensitivity Coefficients

There are three ways to determine sensitivity coefficients:

- Mathematically
- Numerically
- Experimentally

$$c_i = \frac{\partial f}{\partial x_i}$$

Recap – Measurement Functions



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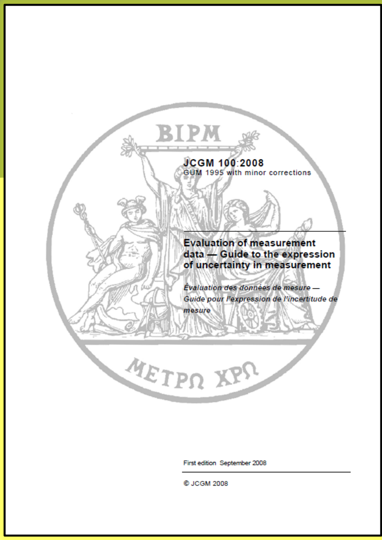
The Law of Propagation of Uncertainties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Adding in quadrature

Bit that deals with **error correlation**

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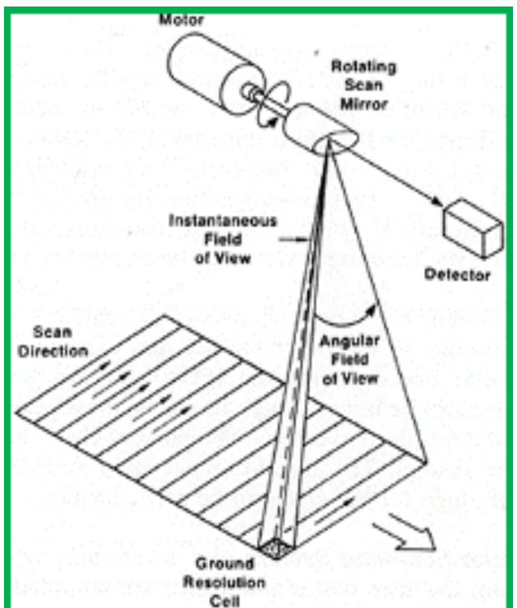
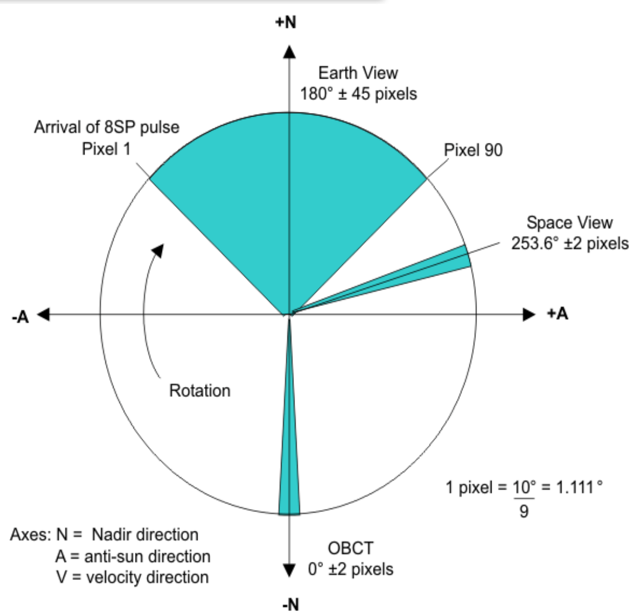
Error Correlation

ERROR *is* NOT *the* same *as* Uncertainty

Uncertainties are never systematic/random
Uncertainties cannot be correlated

Errors can be systematic/random
Errors can be correlated





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Review – key concepts

- ERROR *is* **NOT** *the* same *as* Uncertainty
- Standard uncertainties are standard deviations of the distribution; expanded uncertainties have a coverage factor to reach a probability level (e.g. $k = 2$)
- Measurements are rarely direct: usually the measurand is calculated using a measurement function
- The GUM describes how to propagate uncertainties through a measurement function
- Uncertainties cannot be correlated; but errors can!