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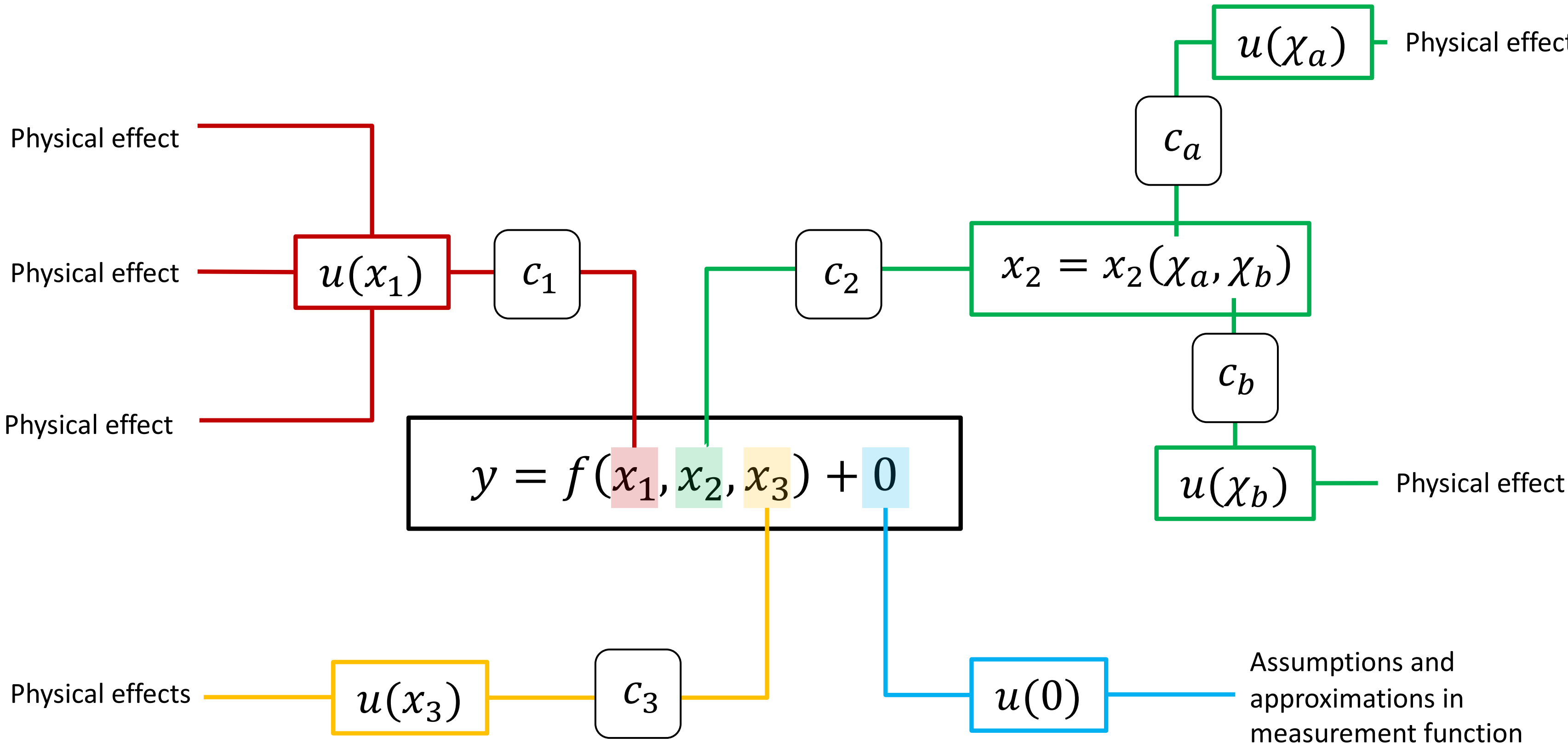
FIDUCEO workshop Lisbon 17-19th April 2018

Uncertainty Concepts 2:

Error Correlation & Covariance

Recap – Measurement Functions



Recap – Law of the Propagation of Uncertainties

$$u_c^2(y) = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j)$$

- $u_c^2(y)$ – Combined uncertainty of measurand
- $u^2(x_i)$ – Uncertainty of each input quantity
- c_i – Sensitivity of the measurand to the input quantity
- $u(x_i, x_j)$ – Covariance of input quantities x_i and x_j

(Much simpler matrix form coming later….)

Covariance

Measure of the joint variability of two variables,

$$u(x_i, x_j) = r_{ij}\, u(x_i)u(x_j)$$

Covariance

Measure of the joint variability of two variables,

$$u(x_i, x_j) = r_{ij}\, u(x_i) u(x_j)$$

- r_{ij} – error correlation coefficient between input quantities x_i and x_j

Error Correlation Structures: Independent Errors

If the errors of measured values x_i and x_j are entirely independent of each other,

$$r_{ij} = 0\,,$$

so that,

$$u(x_i, x_j) = 0$$

Example: Two separate sensor measurements of Earth counts - subject to random noise

Error Correlation Structures: Common Errors

If the errors of measured values x_i and x_j are entirely common to each other,

$$r_{ij} = 1,$$

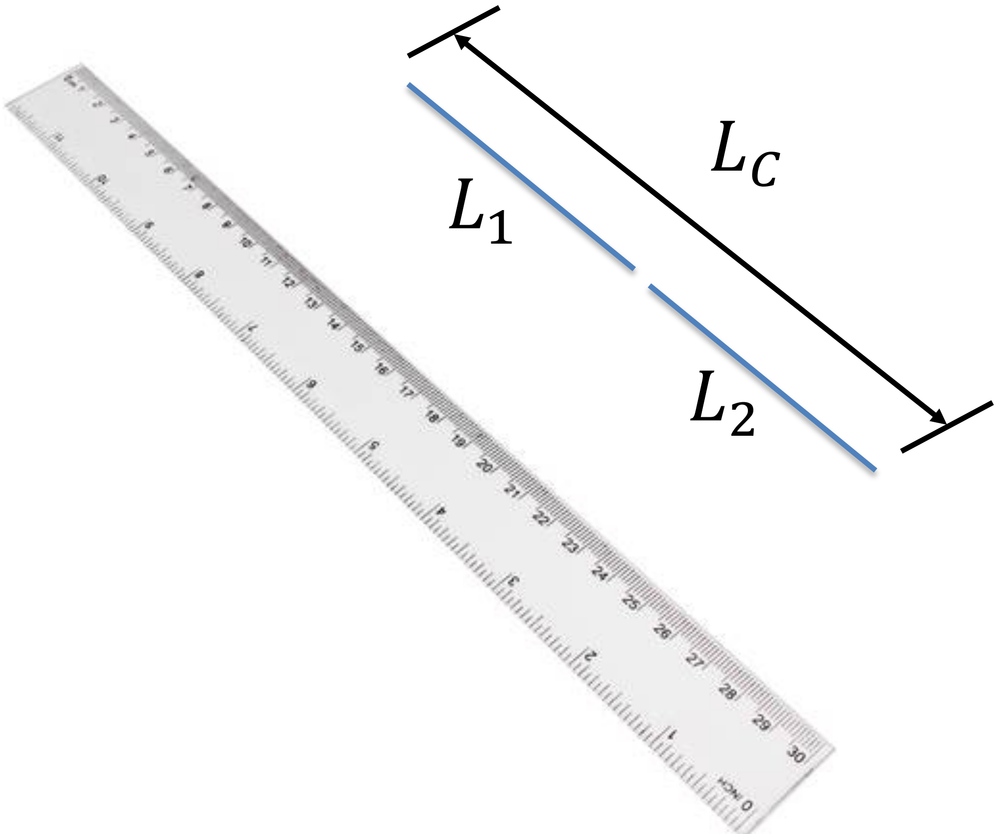
so that,

$$u(x_i, x_j) = u(x_i)u(x_j)$$

Example: Systematic error of thermometer measuring the temperature of internal blackbody calibration target

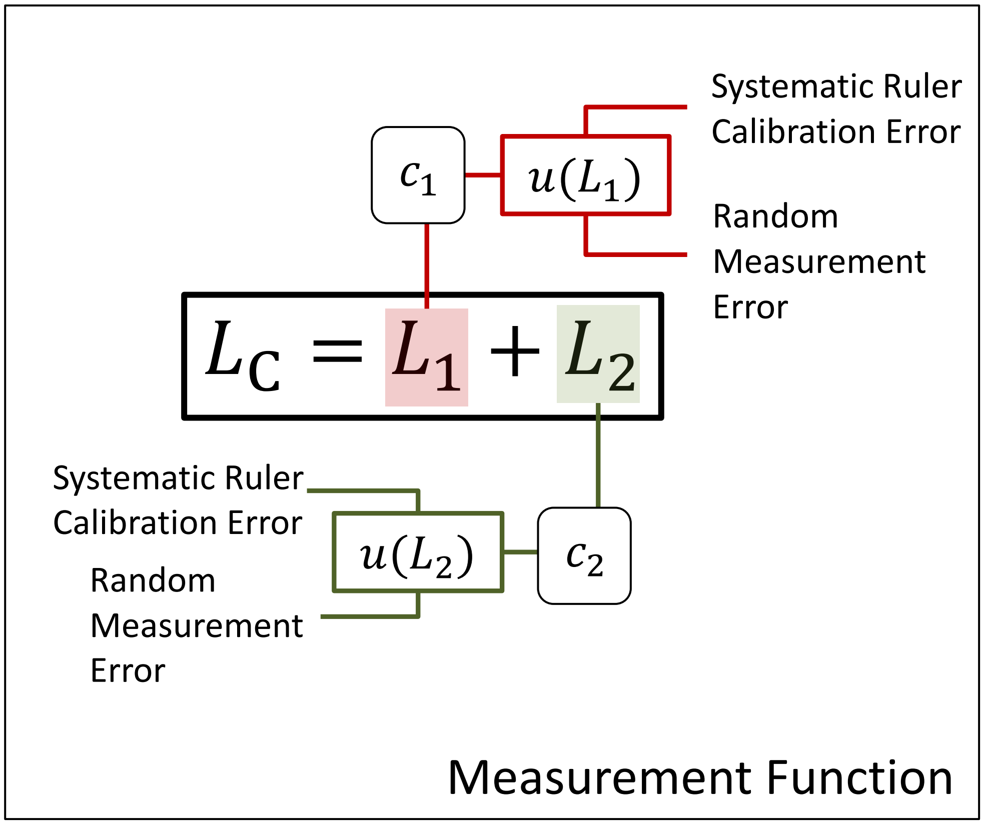
Uncertainty Propagation Example

Example: Combine Two Distance Measurements



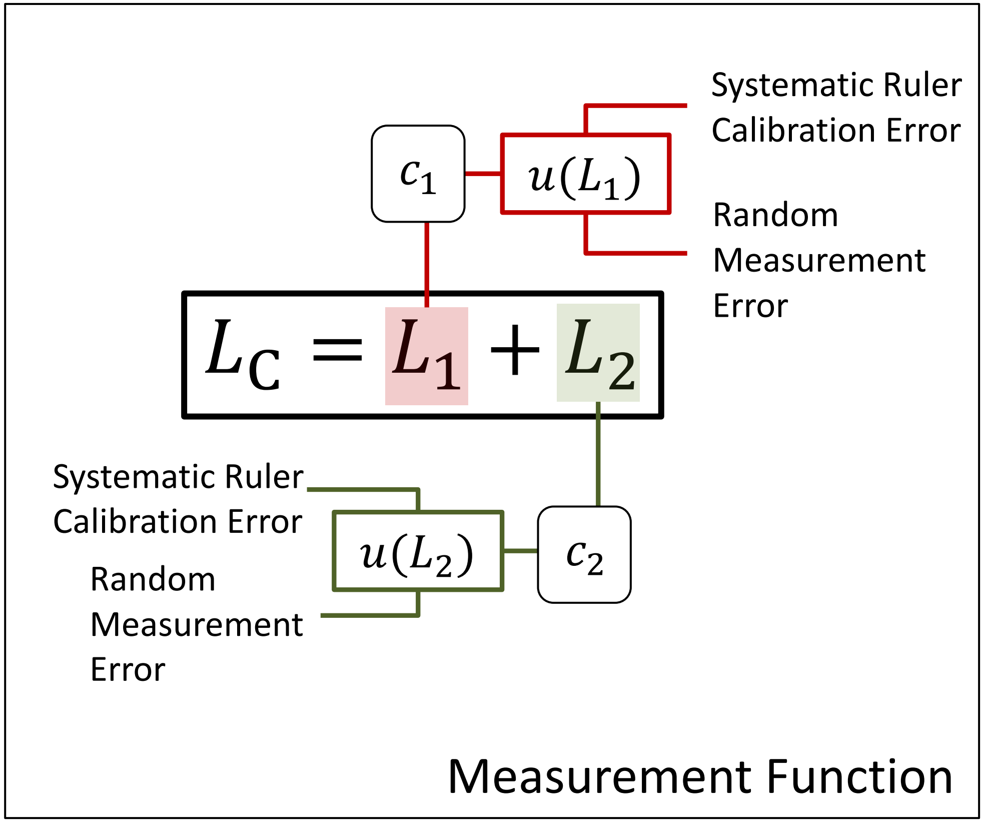
Uncertainty Propagation Example

Example: Combine Two Distance Measurements



Uncertainty Propagation Example

Example: Combine Two Distance Measurements

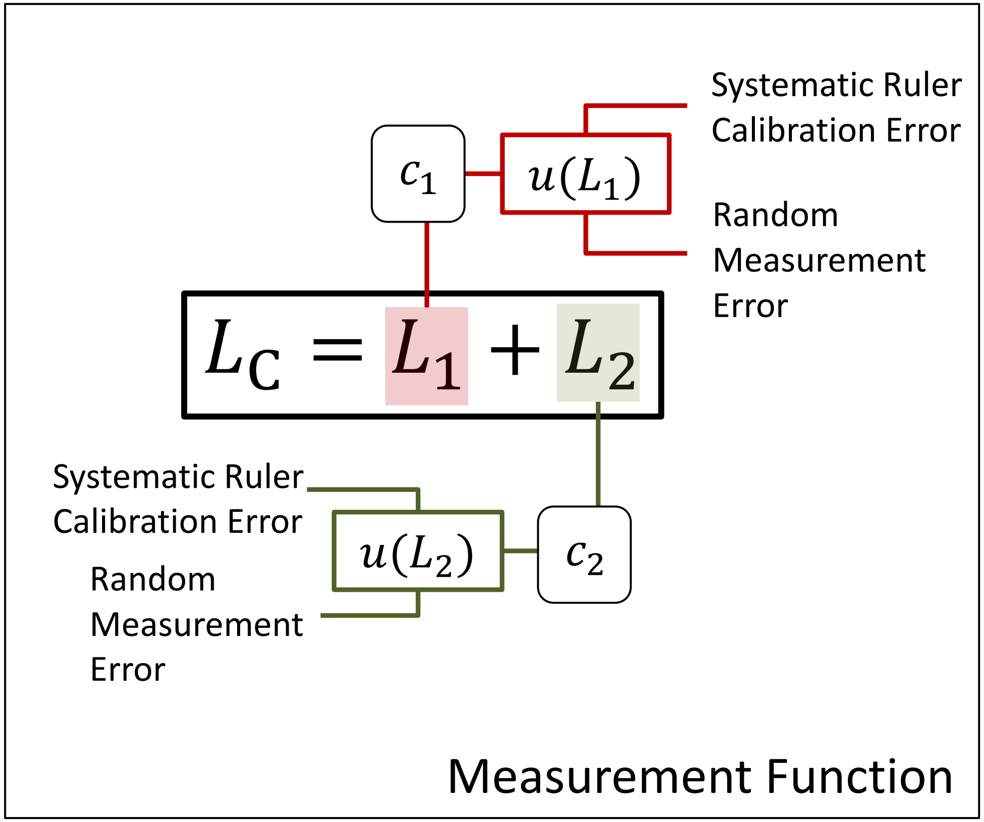


Uncertainties

- Random Error, → uncertainty - $u_R(T_i) = 0.05$ cm
- Systematic Error, → uncertainty - $u_S(T_i) = 0.1$ cm

Uncertainty Propagation Example

Example: Combine Two Distance Measurements

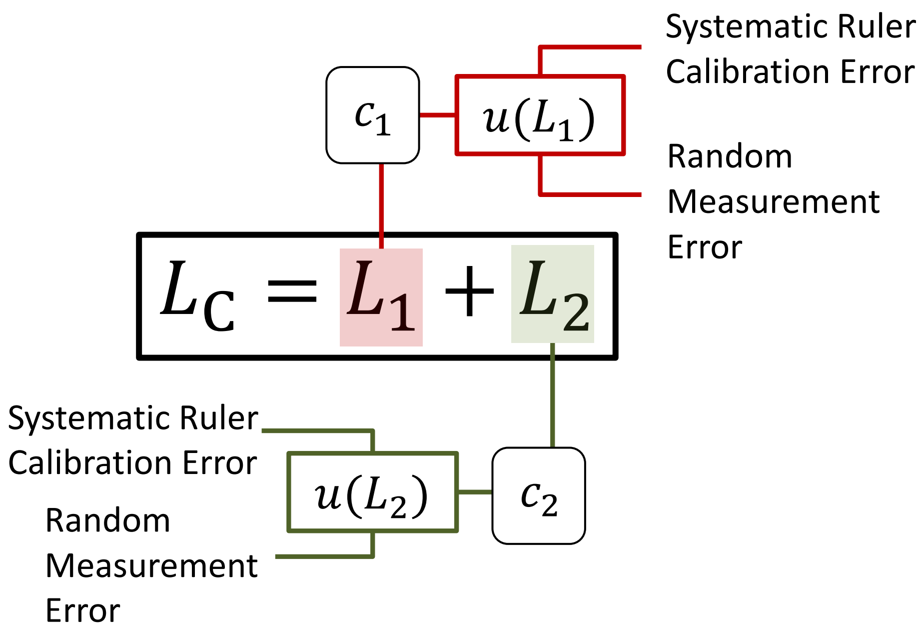


Sensitivity Coefficients

- $c_1 = \frac{\partial L_C}{\partial L_1} = 1$
- $c_2 = \frac{\partial L_C}{\partial L_2} = 1$

Uncertainty Propagation Example

Example: Combine Two Distance Measurements



Measurement Function

Correlation

Same ruler used so:

- Random uncertainty - independent errors –
  $r_{r,12} = 0$
- Systematic uncertainty - common error –
  $r_{s,12} = 1$

Uncertainty Propagation Example

Example: Combine Two Distance Measurements

Evaluate uncertainty

- Due to random effect – with $r_{12} = 0$

$$u_{\mathrm{R}}^2(T_{\mathrm{C}}) = c_1^2 u_{\mathrm{R}}^2(T_1) + c_2^2 u_{\mathrm{R}}^2(T_2)$$

- Due to systematic effect – with $r_{12} = 1$

$$u_{\mathrm{S}}^2(T_{\mathrm{C}}) = c_1^2 u_{\mathrm{S}}^2(T_1) + c_2^2 u_{\mathrm{S}}^2(T_2) + 2c_1 c_2 u_{\mathrm{S}}(T_1) u_{\mathrm{S}}(T_2)$$

Uncertainty Propagation Example

Example: Combine Two Distance Measurements

Evaluate uncertainty

Since $c_1 = c_2 = 1$, combined,

$$u^2(T_\mathrm{C}) = u_\mathrm{R}^2(T_1) + u_\mathrm{R}^2(T_2) + u_\mathrm{S}^2(T_1) + u_\mathrm{S}^2(T_2) + 2u_\mathrm{S}(T_1)u_\mathrm{S}(T_2)$$

Uncertainty Propagation Example

Example: Combine Two Distance Measurements

Evaluate uncertainty

Evaluate,

$$u(T_C) = \sqrt{0.05^2 + 0.05^2 + 0.1^2 + 0.1^2 + 2 \cdot 0.1 \cdot 0.1} \approx 0.21 \text{ cm}$$

Uncertainty Propagation Example

Example: Add Two Thermometer Measurements

Evaluate uncertainty

Evaluate,

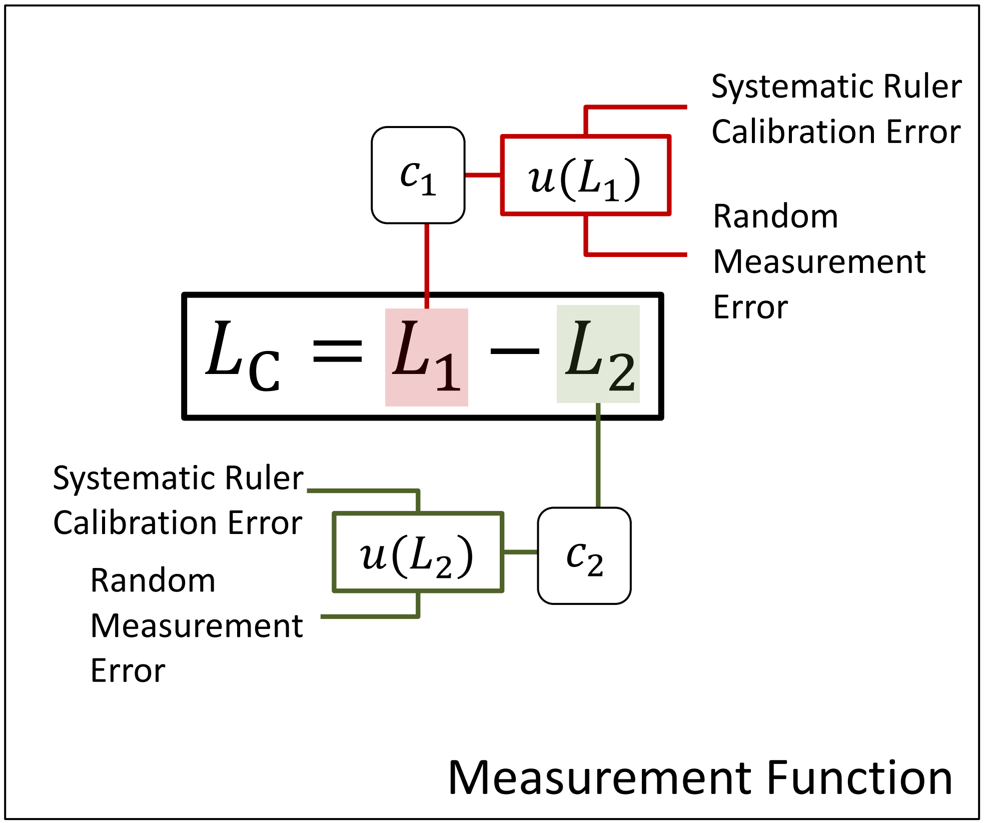
$$u(T_C) = \sqrt{0.05^2 + 0.05^2 + 0.1^2 + 0.1^2 + 2 \cdot 0.1 \cdot 0.1} \approx 0.21 \text{ cm}$$

What if I missed out covariance?

$$u(T_C) = \sqrt{0.05^2 + 0.05^2 + 0.1^2 + 0.1^2} \approx 0.16 \text{ cm !}$$

Uncertainty Propagation Example

Example: Subtract Two Distance Measurements



Sensitivity Coefficients

- $c_1 = \frac{\partial L_C}{\partial L_1} = 1$
- $c_2 = \frac{\partial L_C}{\partial L_2} = -1$

Uncertainty Propagation Example

Example: Subtract Two Distance Measurements

Evaluate uncertainty

Combined uncertainty,

$$u^2(T_\mathrm{C}) = u_\mathrm{R}^2(T_1) + u_\mathrm{R}^2(T_2) + u_\mathrm{S}^2(T_1) + u_\mathrm{S}^2(T_2) - 2u_\mathrm{S}(T_1)u_\mathrm{S}(T_2)$$

Uncertainty Propagation Example

Example: Add Two Thermometer Measurements

Evaluate uncertainty

Evaluate,

$$u(T_C) = \sqrt{0.05^2 + 0.05^2 + 0.1^2 + 0.1^2 - 2 \cdot 0.1 \cdot 0.1} \approx 0.07 \text{ cm}$$

Without covariance

$$u(T_C) = \sqrt{0.05^2 + 0.05^2 + 0.1^2 + 0.1^2} \approx 0.16 \text{ cm}$$

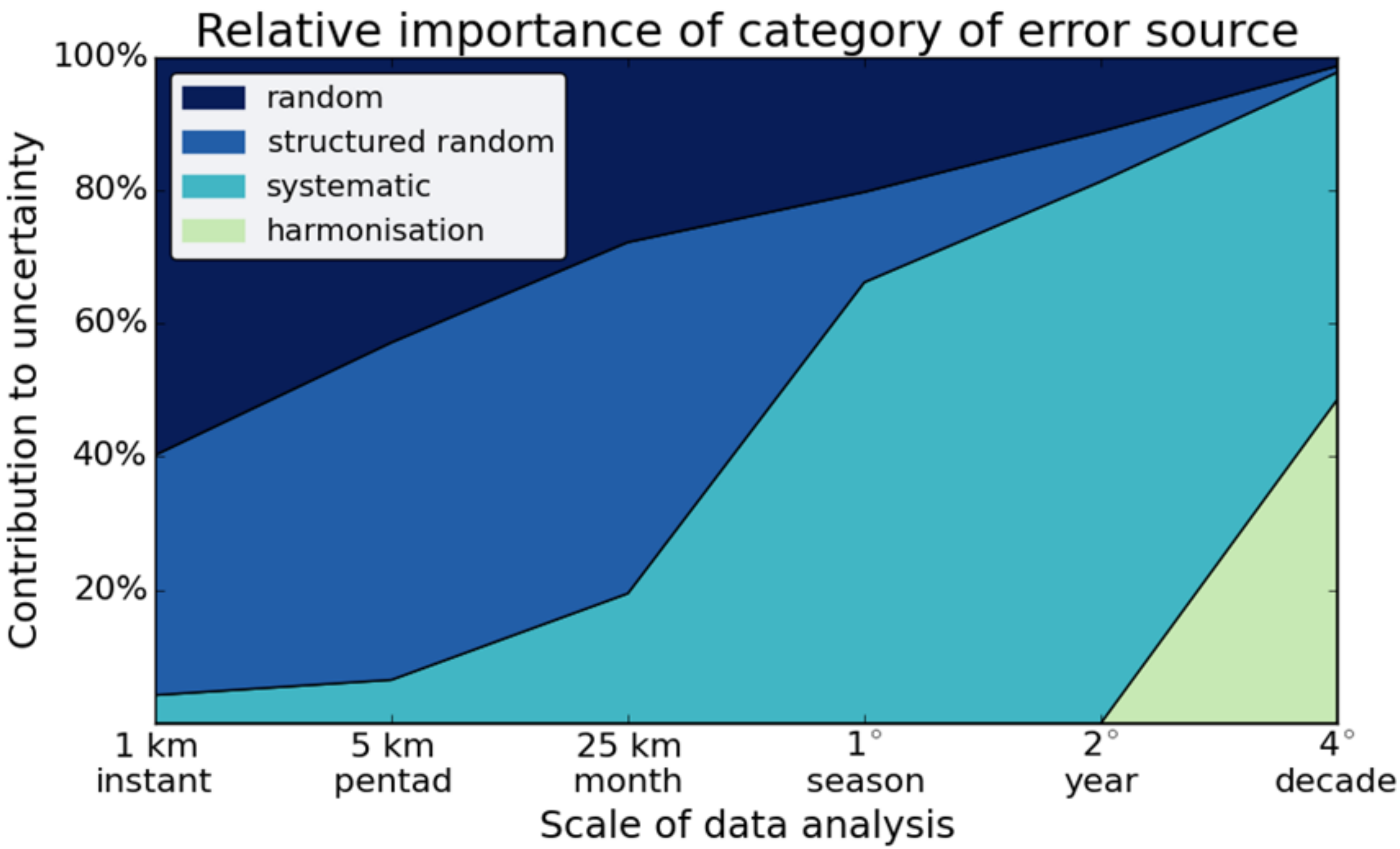
Error Correlation Structures: Structured Correlation

Intermediate forms of error correlation also important.

Very common in Earth Observation datasets with functional forms across different scales, i.e. between pixels:

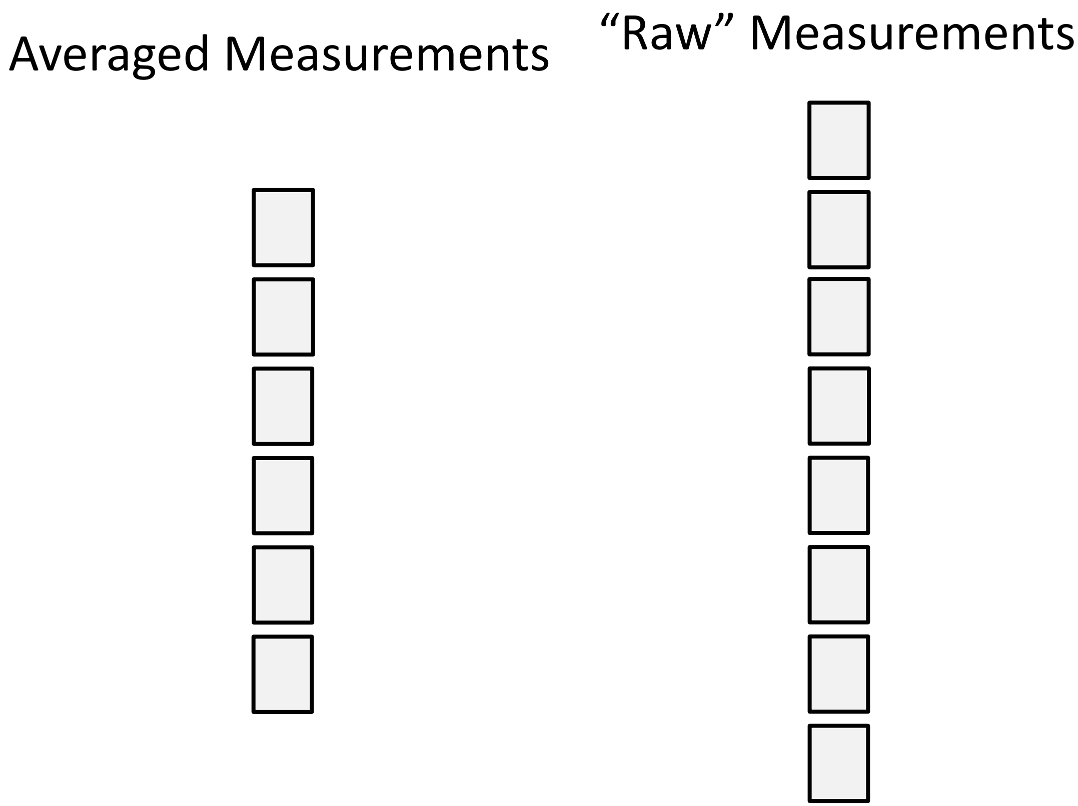
- On a scanline
- Between scanlines
- Between orbit to orbit
- Over the lifetime of a mission
- Between spectral channels

Error Correlation Structures: Why do we care?



Error Correlation Example

Example: Error Correlation between Averaged Measurement Series

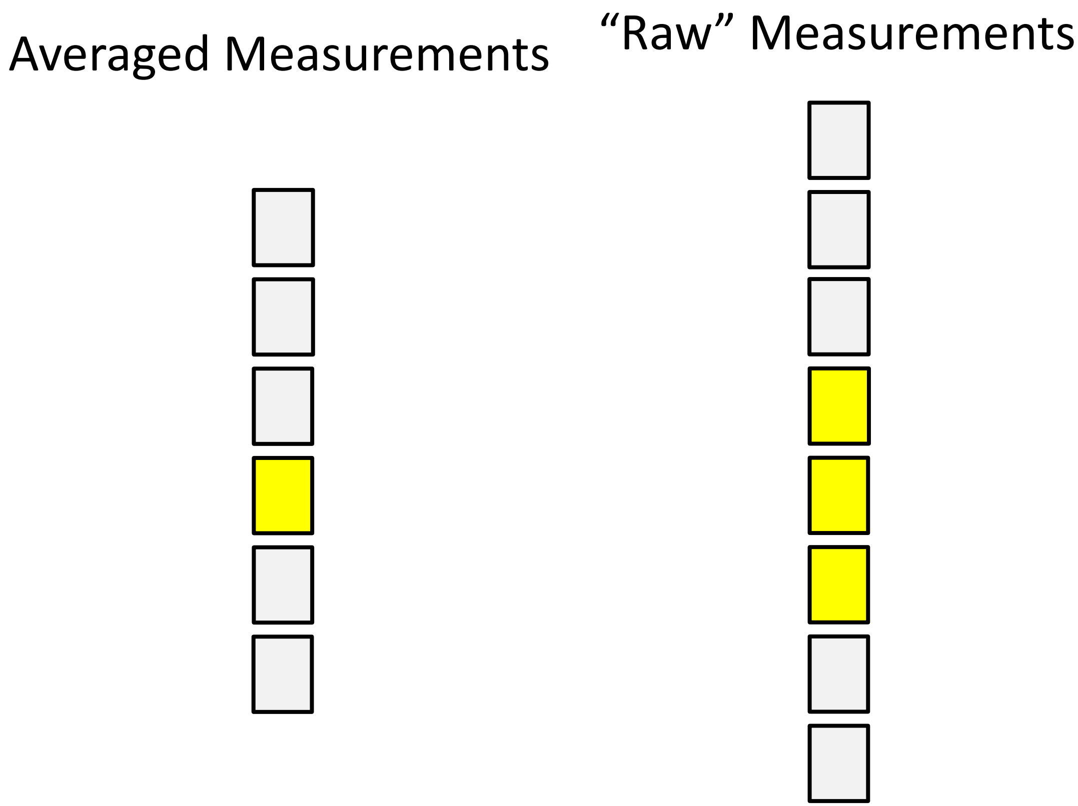


Error Correlation Example

Example: Error Correlation between Averaged Measurement Series

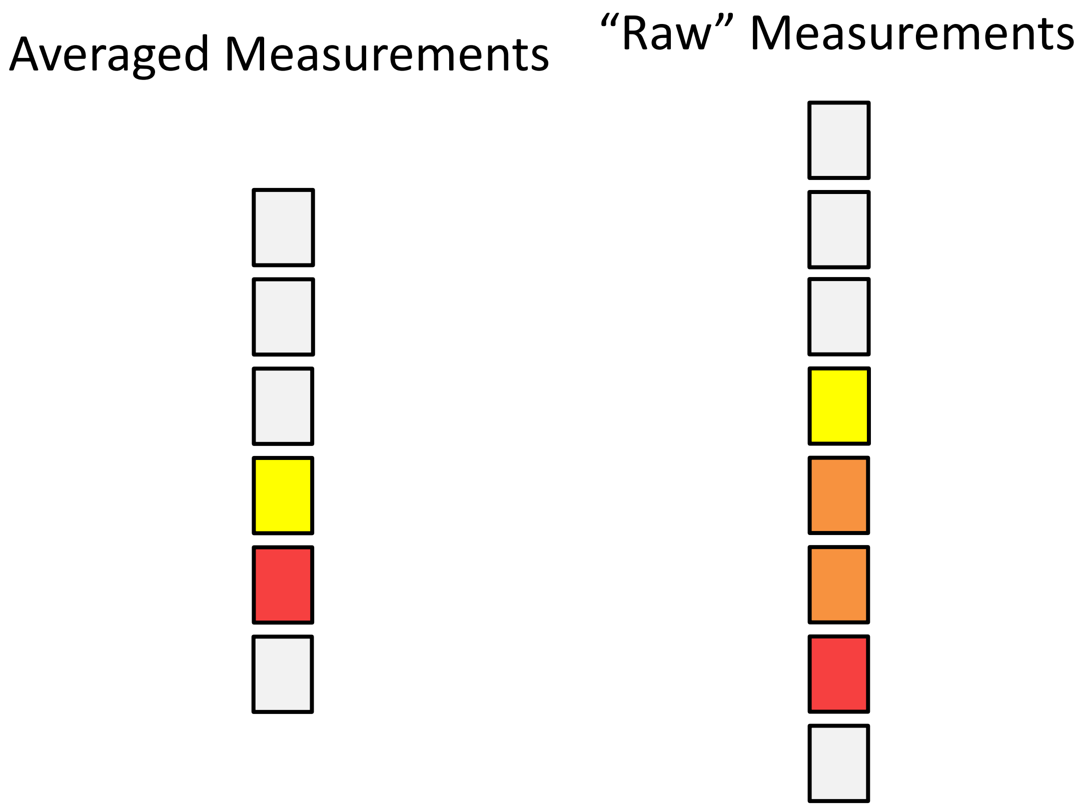
Error Correlation Example

Example: Error Correlation between Averaged Measurement Series



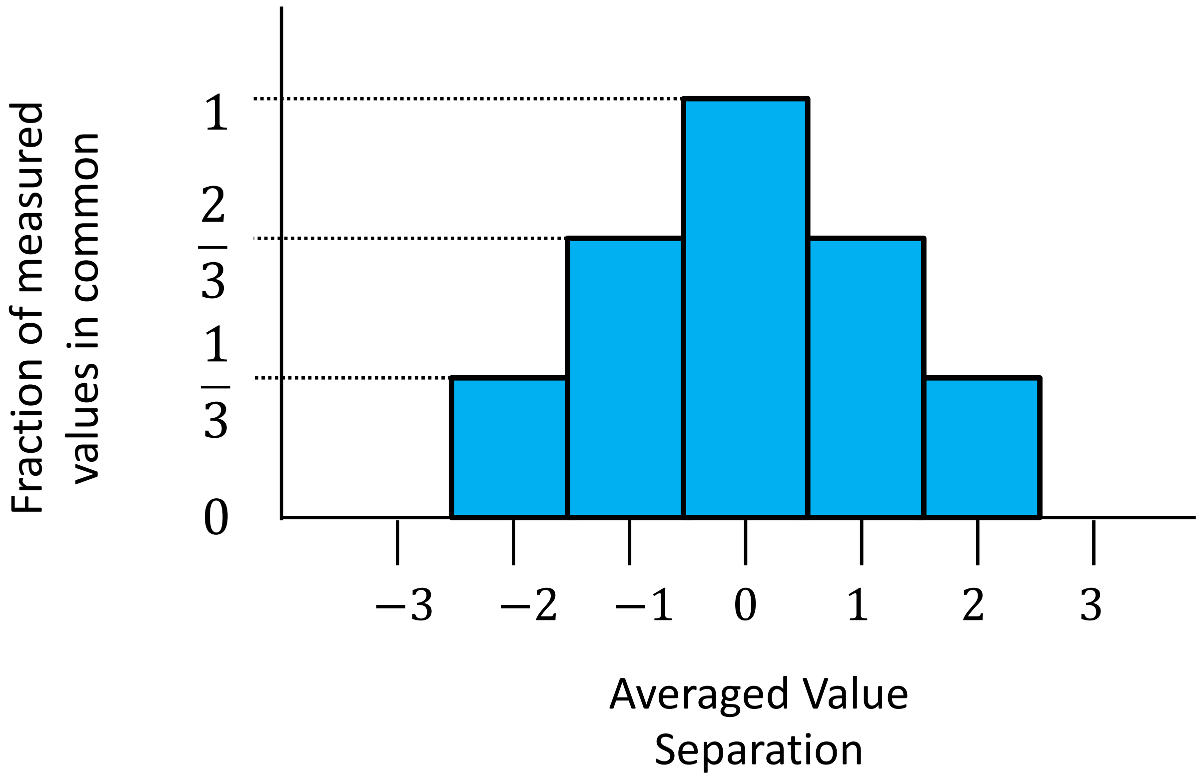
Structured Random Error Correlation

Example: Error Correlation between Averaged Measurement Series



Error Correlation Example

Example: Error Correlation between Averaged Measurement Series



FIDUCEO Term

Triangle Relative Correlation

Error Correlation Structures – FIDUCEO Definitions

- Rectangle Absolute
  - Fully systematic or systematic within a calibration period
- Triangle Relative
  - Rolling averages
- Bell-shaped Relative
  - Weighted rolling averages, splines, smoothing, other
- Repeating
  - E.g. once per orbit, diurnal or seasonal cycles
- Mixed

Error Correlation Structures – Reporting

Table descriptor		Comments	Example
Name of effect		A unique name	Internal calibration target count noise
Affected term in measurement function		Name and standard symbol	
Instruments in the series affected		Identifier	All instruments all satellites
Correlation type and form	Pixel-to-pixel [pixels]	One of the types	Rectangular absolute
	from scanline to scanline [scanlines]		Triangular relative
	between images [images]		N/A for orbiting satellite
	Between orbits [orbit]		Random
	Over time [time]		Random
Correlation scale	Pixel-to-pixel [pixels]	As needed to define type	[-∞,∞] (fully correlated across scan)
	from scanline to scanline [scanlines]		n = 51 (51 scanlines averaged in rolling average)
	between images [images]		N/A for orbiting satellite
	Between orbits [orbit]		0
	Over time [time]		0
Channels/bands	List of channels / bands affected	Channel names	All channels
	Error correlation coefficient matrix	A matrix	Identity matrix (diagonal).
Uncertainty	PDF shape	Functional form	Gaussian
	units	Units	Counts
	magnitude		Given once per orbit file
Sensitivity coefficient		Value, equation or parameterisation of sensitivity of measurand to term	

Summarising

- Error covariance: joint variability of two quantities due to common errors
- Error correlation can increase or decrease overall uncertainty (depending whether common errors cancel out)
- Structured random effects create error correlations that FIDUCEO categories into correlation forms
- FIDUCEO uses an effects table to consider correlation structures and provide uncertainty information