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Metrological Concepts 3: measurement function uncertainty analysis

The Measurement Function

- The measurement function converts the observed quantities into the measurand

$$L_E = a_0 + \frac{a_1 L_T - a_2 C_T^2}{C_T} C_E + a_2 C_E^2 + 0$$

(AVHRR like example)

- Note that in FIDUCEO we try and ensure that we understand the uncertainties associated with each term but also have a physically based understanding of all components of the equation

The Measurement Function

Calibration Parameters

Calibration Target Counts

Earth Counts

$$L_E = a_0 + \frac{a_1 L_T - a_2 C_T^2}{C_T} C_E + a_2 C_E^2 + 0$$

Radiance of calibration target

The Measurement Function

Instrument Gain term. For the AVHRR especially Impacted by errors in L_T

Assumptions and approximations in measurement function

$$L_E = a_0 + \frac{a_1 L_T - a_2 C_T^2}{C_T} C_E + a_2 C_E^2 + 0$$

Bias term. May be related to difference is the view of the instrument between Earth view and Space View for example. May be a function of time/instrument temperature

Non-linear term. Assumes a quadratic.

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+0 Term

- Appears in a number of places
- Intended to force investigation of assumptions
  - To think about and characterize known unknowns
  - Mostly via Type-B uncertainty estimates
- Not measured but via expert knowledge (including modelling)
  - AVHRR examples
    - Quadratic assumption for non-linearity effect
    - Constant non-linear coefficient
    - Numerical issues (digitisation/numerical integration)
    - Etc.

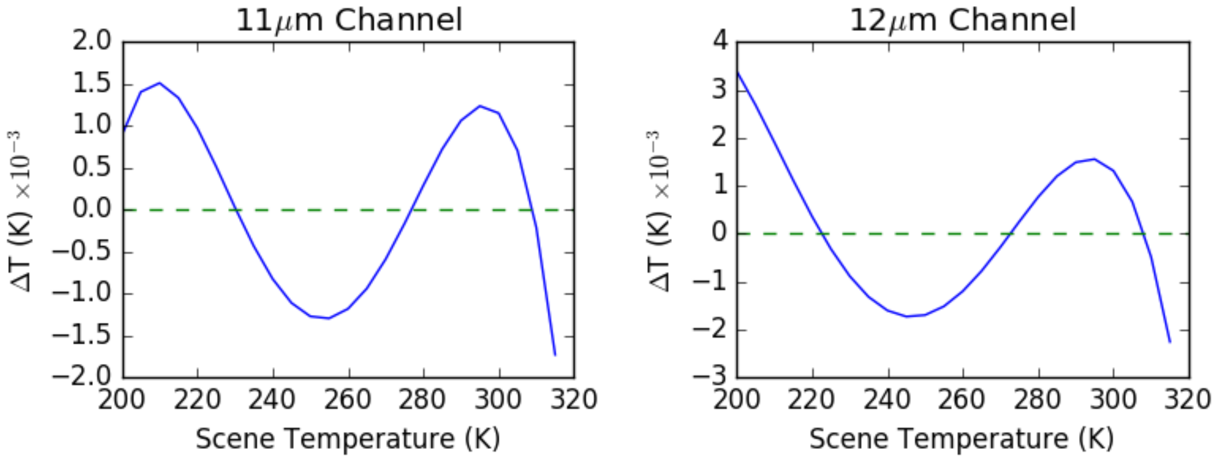
HgCdTe Non-Linearity

- As an example – how good is a quadratic model to describe non-linear detectors?

Simulation of HgCdTe detector comparing response with a quadratic function

Shows error by assuming quadratic likely to be small

This needed to be done to make sure we understand the importance of different sources of uncertainty



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Uncertainty Tree

- Given the measurement equation we then need to investigate all the possible sources of uncertainty that feed into the final total
- Involves thinking about each term in the measurement function and breaking it down into all sub-processes down to the originating process for uncertainty
- We represent this process by an Uncertainty Tree Diagram

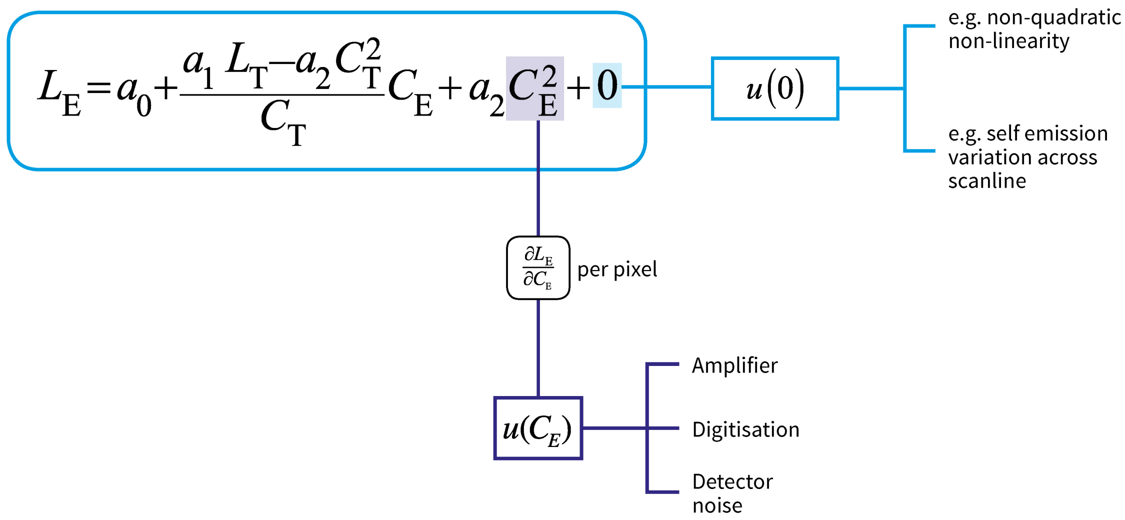
$$L_E = a_0 + \frac{a_1 L_T - a_2 C_T^2}{C_T} C_E + a_2 C_E^2 + 0$$

$$L_{\mathrm{E}} = a_0 + \frac{a_1 L_{\mathrm{T}} - a_2 C_{\mathrm{T}}^2}{C_{\mathrm{T}}} C_{\mathrm{E}} + a_2 C_{\mathrm{E}}^2 + 0$$

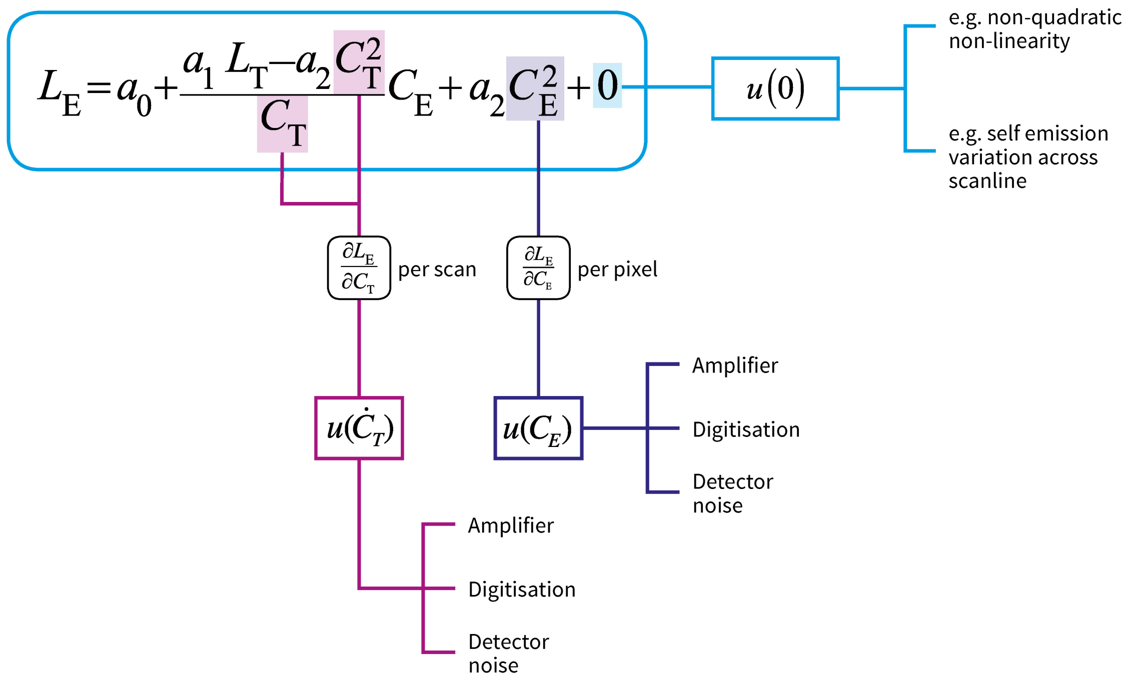
$u(0)$

e.g. non-quadratic non-linearity

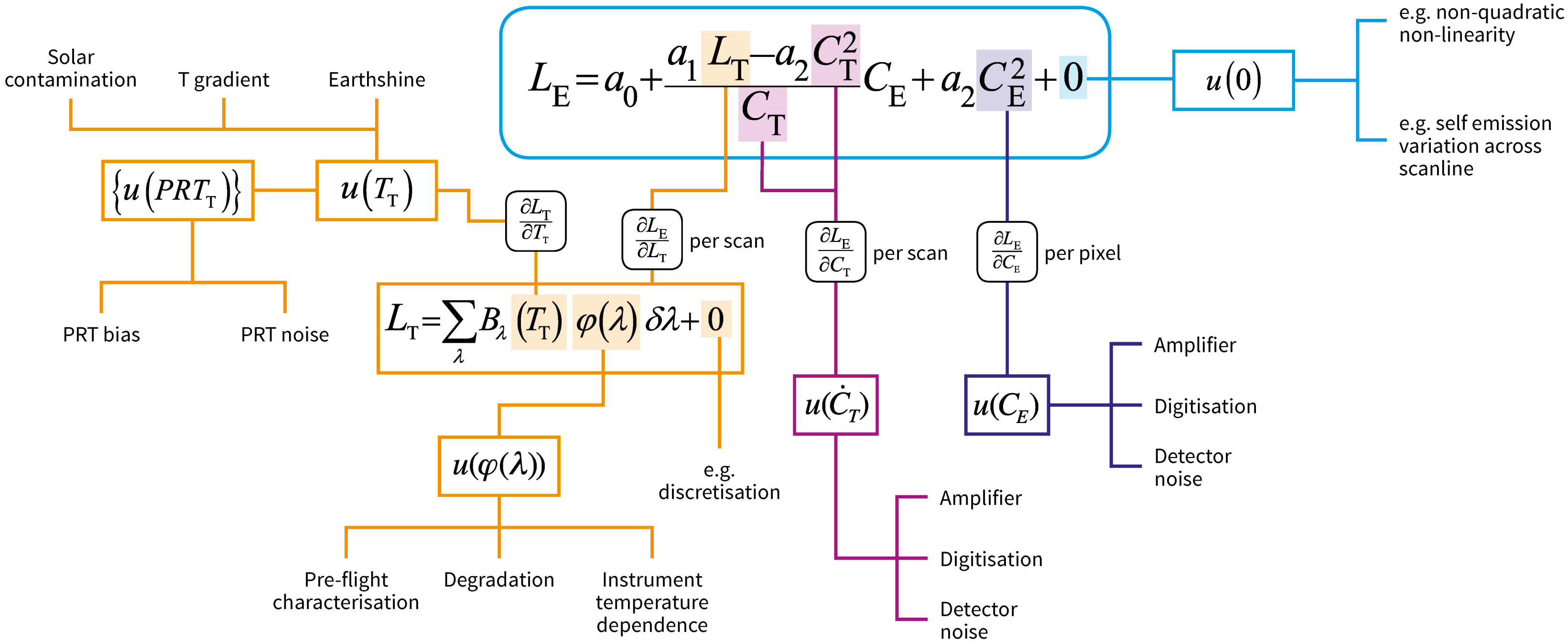
e.g. self emission variation across scanline



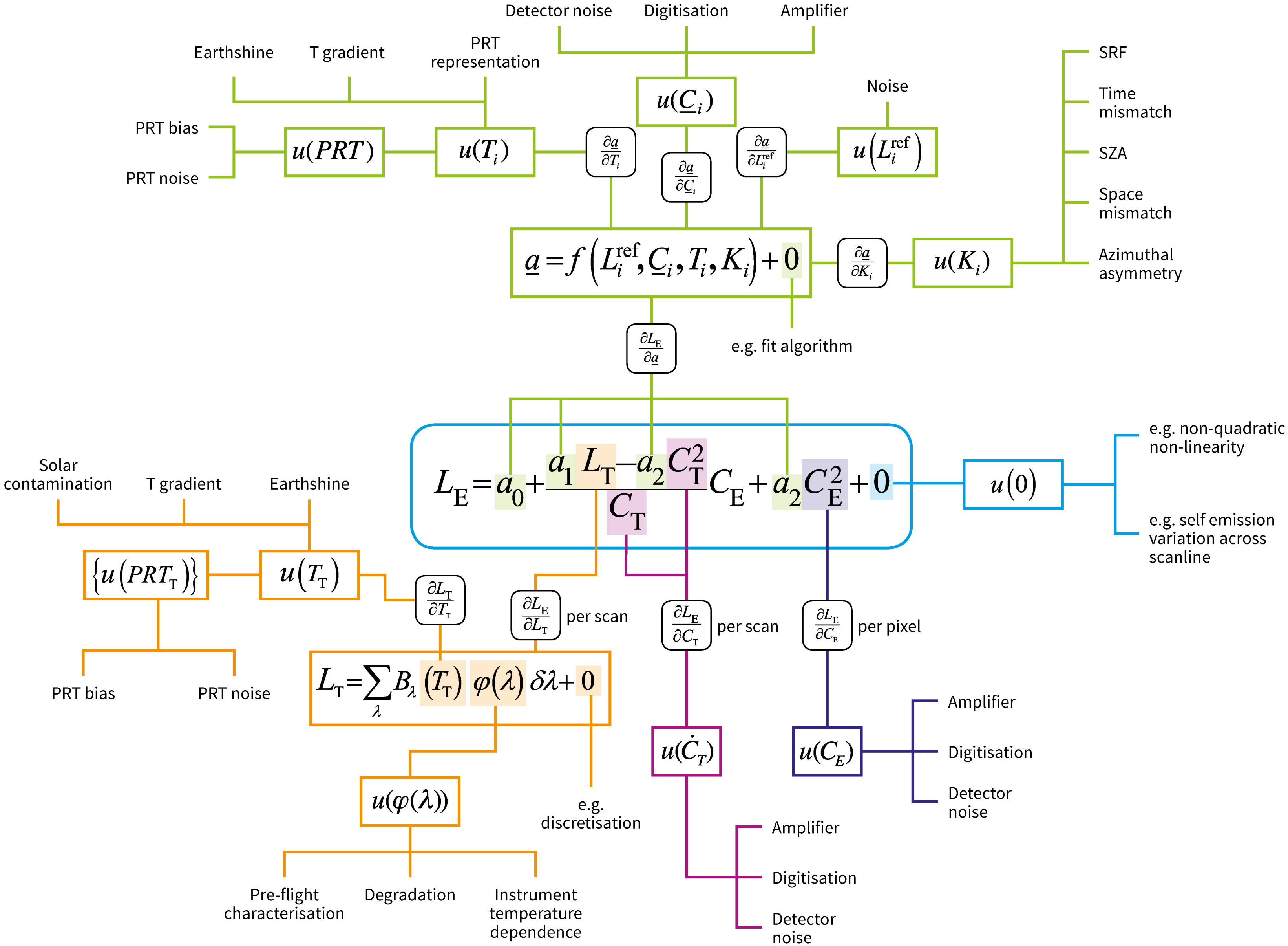
$L_E = a_0 + \frac{a_1 L_T - a_2 C_T^2}{C_T} C_E + a_2 C_E^2 + 0$
$u(0)$
e.g. non-quadratic non-linearity
e.g. self emission variation across scanline
$\frac{\partial L_E}{\partial C_E}$
per pixel
$u(C_E)$
Amplifier
Digitisation
Detector noise



$L_E = a_0 + \frac{a_1 L_T - a_2 C_T^2}{C_T} C_E + a_2 C_E^2 + 0$
$u(0)$
e.g. non-quadratic non-linearity
e.g. self emission variation across scanline
$\frac{\partial L_E}{\partial C_T}$
per scan
$\frac{\partial L_E}{\partial C_E}$
per pixel
$u(\dot{C}_T)$
$u(C_E)$
Amplifier
Digitisation
Detector noise
Amplifier
Digitisation
Detector noise



$L_E = a_0 + \frac{a_1 L_T - a_2 C_T^2}{C_T} C_E + a_2 C_E^2 + 0$
$u(0)$
e.g. non-quadratic non-linearity
e.g. self emission variation across scanline
Solar contamination
T gradient
Earthshine
$\{u(PRT_T)\}$
$u(T_T)$
PRT bias
PRT noise
$\frac{\partial L_T}{\partial T_T}$
$\frac{\partial L_E}{\partial L_T}$ per scan
$\frac{\partial L_E}{\partial C_T}$ per scan
$\frac{\partial L_E}{\partial C_E}$ per pixel
$L_T = \sum_\lambda B_\lambda(T_T)\varphi(\lambda)\delta\lambda + 0$
$u(\varphi(\lambda))$
e.g. discretisation
Pre-flight characterisation
Degradation
Instrument temperature dependence
$u(\dot{C}_T)$
Amplifier
Digitisation
Detector noise
$u(C_E)$
Amplifier
Digitisation
Detector noise



Detector noise
Digitisation
Amplifier
Earthshine
T gradient
PRT representation
$u(C_i)$
Noise
SRF
Time mismatch
PRT bias
PRT noise
$u(PRT)$
$u(T_i)$
$\frac{\partial a}{\partial T_i}$
$\frac{\partial a}{\partial C_i}$
$\frac{\partial a}{\partial L_i^{\mathrm{ref}}}$
$u\left(L_i^{\mathrm{ref}}\right)$
SZA
Space mismatch
$a = f\left(L_i^{\mathrm{ref}}, C_i, T_i, K_i\right) + 0$
$\frac{\partial a}{\partial K_i}$
$u(K_i)$
Azimuthal asymmetry
$\frac{\partial L_E}{\partial a}$
e.g. fit algorithm
$L_E = a_0 + \frac{a_1 L_T - a_2 C_T^2}{C_T} C_E + a_2 C_E^2 + 0$
$u(0)$
e.g. non-quadratic non-linearity
e.g. self emission variation across scanline
Solar contamination
T gradient
Earthshine
$\{u(PRT_T)\}$
$u(T_T)$
$\frac{\partial L_T}{\partial T_T}$
$\frac{\partial L_E}{\partial L_T}$
per scan
$\frac{\partial L_E}{\partial C_T}$
per scan
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per pixel
PRT bias
PRT noise
$L_T = \sum_\lambda B_\lambda(T_T)\,\varphi(\lambda)\,\delta\lambda + 0$
$u(\dot{C}_T)$
$u(C_E)$
Amplifier
Digitisation
Detector noise
$u(\varphi(\lambda))$
e.g. discretisation
Amplifier
Digitisation
Detector noise
Pre-flight characterisation
Degradation
Instrument temperature dependence

Source Uncertainties

- 'Branches' of Tree Diagram will always end at a process which itself is a source of uncertainty
- Note about uncertainty - obtaining uncertainties means removing all known sources of systematic error
  - GUM (2008) Section 3.2.4
    - It is assumed that the result of a measurement has been corrected for all recognized significant systematic effects and that every effort has been made to identify such effects.
- You therefore have **to understand the process** that is giving rise to the uncertainty before estimating them and **understand any assumptions that are being made**

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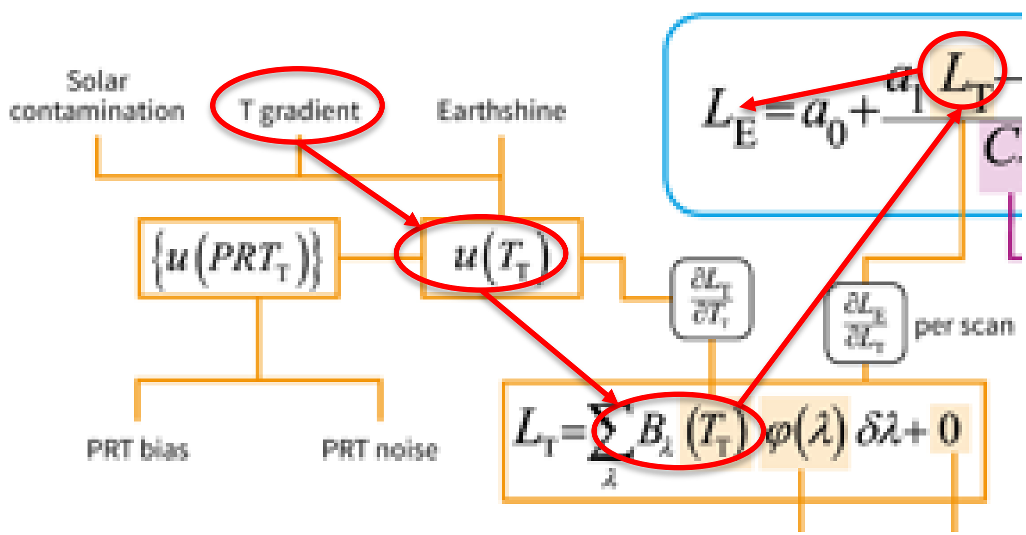
Source uncertainties - example

- Calibration Target temperature
  - Structured uncertainty component
  - Arises from errors in mapping estimates of calibration target temperature to calibration target radiances
  - Propagates through Planck function/spectral response function convolution to an uncertainty on the ICT radiance to a Gain uncertainty to the final Earth radiance
  - Uses the sensitivity coefficients to propagate uncertainties to the measurand



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Assumptions relating to Calibration Target Temperature

- There are a **number of assumptions**
  - **Regarding the temperature distribution across the calibration target**
  - Regarding the location of the PRT relative to the radiating surface
    - Likely PRT measuring the underside of the target and not the surface temperature
  - Regarding any thermal lag between the location of the PRT and the radiative surface
  - Etc.
- Ideally every assumption should have at least an estimate of an uncertainty associated with it
  - For many likely to be a Type-B estimate (expert opinion)
  - If thought small may be ignored relative to other larger uncertainty components for the same process
- For each effect need to go through a process to understand/estimate it.
  - Here we consider the radiant temperature of the calibration target

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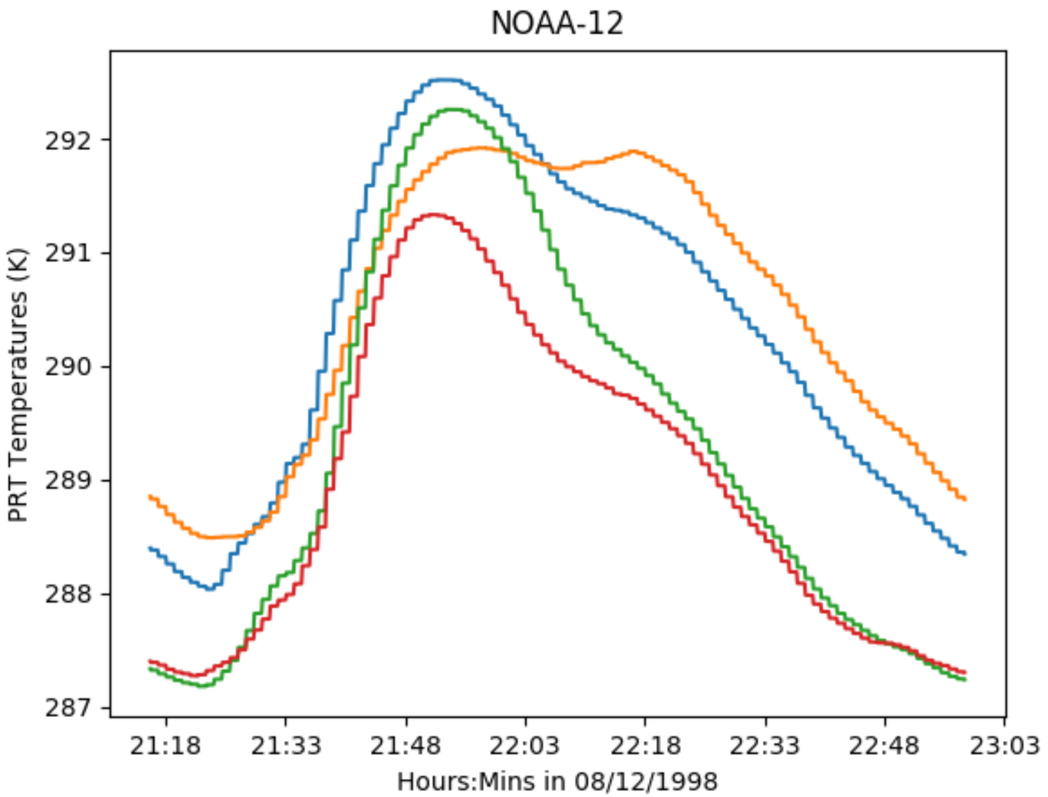
First: Understand the signal

- Need to think about for the process giving rise to the term in the uncertainty tree
  - What is **physically** happening?
    - For the AVHRR calibration target a large (20cm circular) area is being imaged to a single detector
    - Signal is integral of all temperatures across calibration target convolved with the spectral response function to a single radiance
    - Four point measurement of temperature are taken from the calibration target
  - How is it being **modelled** by the calibration scheme?
    - Operational calibration takes average of 4 PRT measurements as estimate of the radiant temperature
  - What **assumptions** are being made?
    - No complex gradients exist across the target and any gradients are planar or flat in nature

Second: Is model of process correct

- Look at PRT data around an orbit

- Large differences between different PRTs indicates strong and changing gradients
- Simple PRT average will introduce errors into the estimate of the calibration target radiance

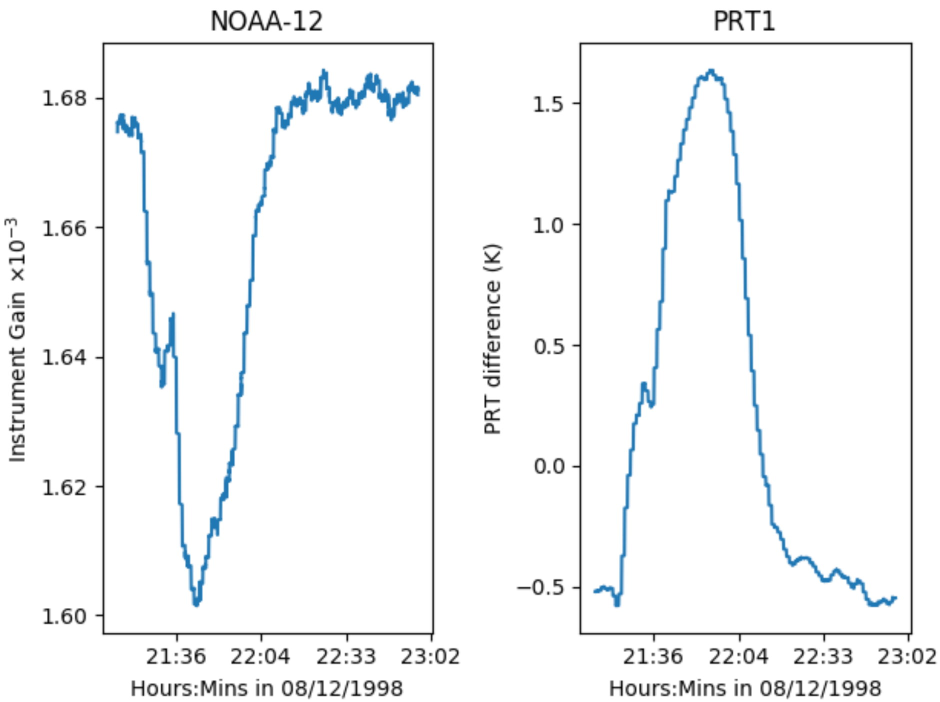


Third: Correct systematic effects as well as can be done

- From the data we have can we estimate the size of the possible error due to gradients?
  - Estimate of possible error from difference from a plane fitted to 3 PRTs when compared to the 4th
    - Indicate possible error relative to taking average of PRTs
    - Have to check each PRT in turn
  - Use knowledge of detector system
    - 3.7µm channel **should be linear** (no gain variation around an orbit)
    - 3.7µm gain variations will indicate something about the temperature error

PRT difference from a Plane

- First estimate of error – difference from a plane for each PRT



Scale of estimated error is large so this error is significant

There is also some symmetry between the estimates

Strong correlation with gain variation

Note that these data are not meant to represent the actual error but are indicators of its scale and time dependence

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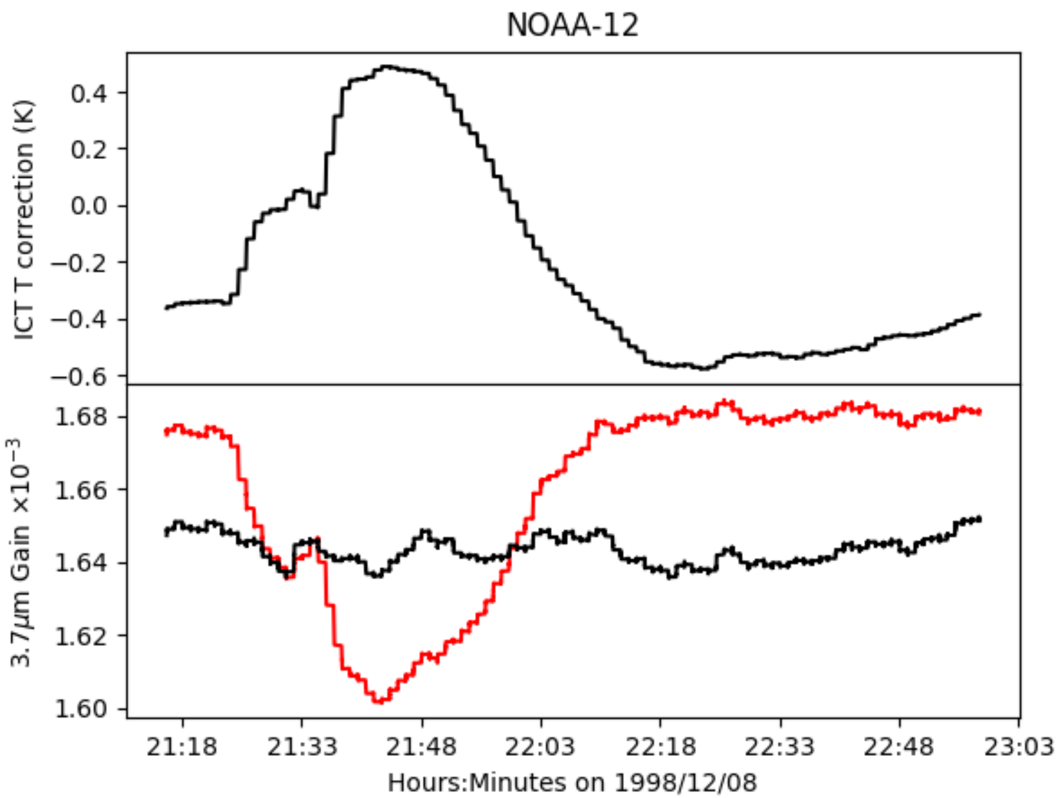
A model for the Calibration Target Temperature Error

- A model can be derived to estimate the size of the error from an understanding of the behaviour of the detection system
  - For the 3.7µm channel the InSb detector is **linear**
  - Only expect to see small (if any) gain variations around an orbit
  - Assume variable gain is caused by the Calibration Target Temperature Error
  - Fit combination of PRT error curves to minimise the 3.7µm gain

$$T_T^{Error} = b_1 PRT_1^{Err} + b_2 PRT_2^{Err} + b_3 PRT_3^{Err} + b_4 PRT_4^{Err}$$

Target Temperature Error estimate

- NOAA-12 example



Top panel shows estimated Calibration Target Temperature error

Lower panel shows the original 3.7µm gain (red) and corrected (black)

Associated Uncertainty

- An uncertainty needs to be added to the analysis. In this case it is an approximation based on the 3.7µm gain standard deviation
  - Based on inverting band coefficient temperature to radiance equation

$$u(T_T)^2 = \left(\frac{\partial T_T}{\partial L_T}\frac{\partial L_T}{\partial Gain}\right)^2 u(Gain)^2$$

where

$$Gain = \frac{a_1 L_T}{C_T}$$

and

$$L_T = \frac{C_1 \nu_C^3}{\left(e^{\frac{C_2 \nu_C}{(a+bT_T)}} - 1\right)}$$

E.g. uncertainty for case shown = 0.05018K

Summary of process

- By following the process for one of the branches we have
  - Corrected as best we can for an error in the estimate of the calibration target temperature
    - Model derived by
      - Understanding the limitations of the data we have
      - Understanding the physics behind the signal
  - Estimated the remaining uncertainty
    - Based on the statistics of the corrected data
  - **End up with improved data together with an uncertainty that will be propagated to the measurand**

Error Correlations

- One thing that is not well represented by the Traceability Tree Diagram are showing correlated error effects
- Some correlated effects can be seen in the data itself
  - E.g. HIRS correlated counts (Holl et al. 2018)
- Other correlations arise from the Harmonisation process
  - Calibration coefficients are potentially fixed for entire mission lifetime
- Other correlated error effects are related to how the instrument works
  - One example is the uncertainty on the calibration target temperature. The temperature error will exist for all IR channels so is correlated between channels
  - See later presentation where these sorts of correlations will be discussed in more depth

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