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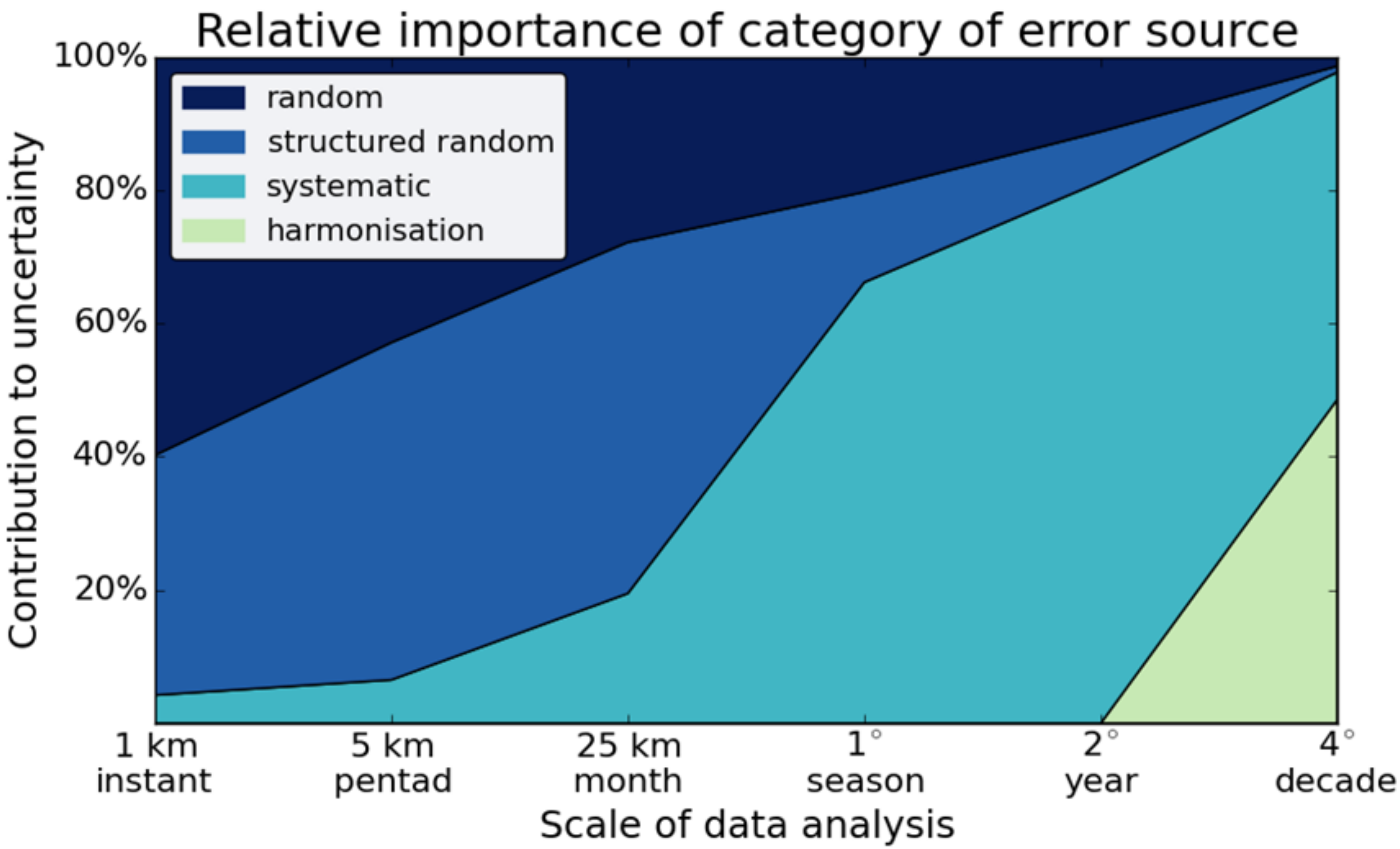
FIDUCEO workshop Lisbon 17-19th April 2018

Uncertainty Concepts 4: Structured Errors & Uncertainty Propagation

Error Correlation Structures – Why do we care?



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Recap: Propagation of Uncertainties

$$u_c^2(y) = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j} u(x_i, x_j)$$

- $u_c^2(y)$ – Combined uncertainty of measurand
- $u^2(x_i)$ – Uncertainty of each input quantity
- c_i – Sensitivity of the measurand to the input quantity
- $u(x_i, x_j)$ – Covariance of input quantities x_i and x_j where,

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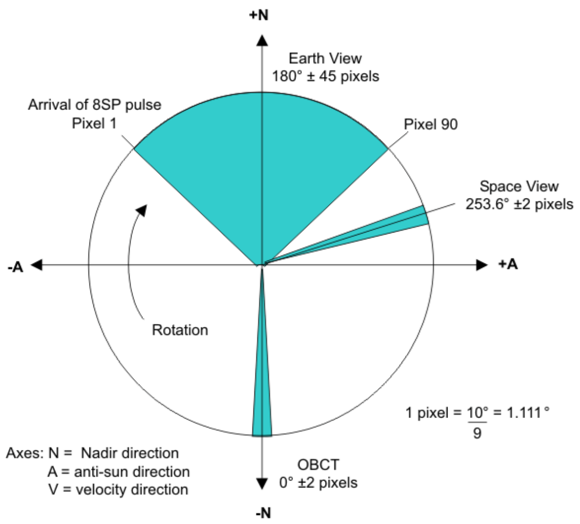
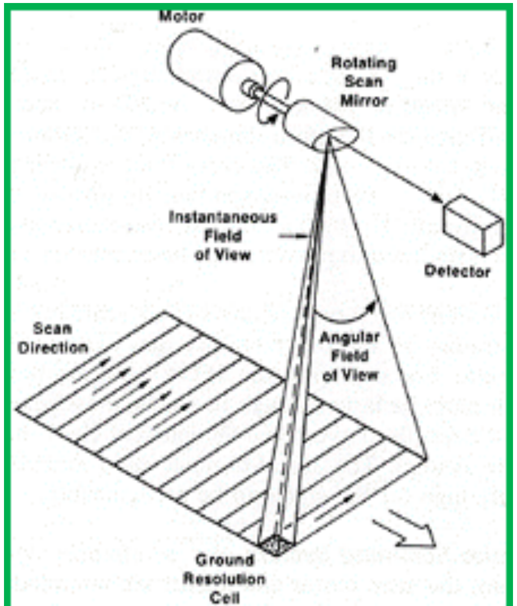
Error Correlation Structures – what dimensions?

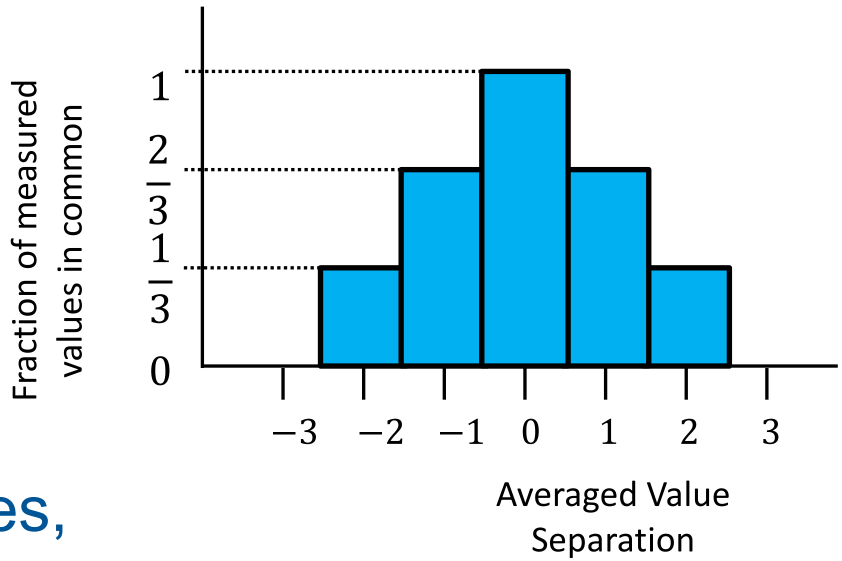
- Spatial/temporal
  - On a scanline
  - In different scanlines
  - From orbit to orbit
  - Over the lifetime of a mission
- Spectral
  - Spectral channels / bands

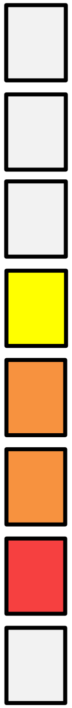




Error Correlation Structures – FIDUCEO Definitions

- Rectangle Absolute
  - Fully systematic or systematic within a calibration period
- Triangle Relative
  - Rolling averages
- Bell-shaped Relative
  - Weighted rolling averages, splines, smoothing, other
- Repeating
  - E.g. once per orbit, diurnal or seasonal cycles
- Mixed





Error Correlation Structures – Reporting

Table descriptor		Comments	Example
Name of effect		A unique name	Internal calibration target count noise
Affected term in measurement function		Name and standard symbol	
Instruments in the series affected		Identifier	All instruments all satellites
Correlation type and form	Pixel-to-pixel [pixels]	One of the types	Rectangular absolute
	from scanline to scanline [scanlines]		Triangular relative
	between images [images]		N/A for orbiting satellite
	Between orbits [orbit]		Random
	Over time [time]		Random
Correlation scale	Pixel-to-pixel [pixels]	As needed to define type	[-∞,∞] (fully correlated across scan)
	from scanline to scanline [scanlines]		n = 51 (51 scanlines averaged in rolling average)
	between images [images]		N/A for orbiting satellite
	Between orbits [orbit]		0
	Over time [time]		0
Channels/bands	List of channels / bands affected	Channel names	All channels
	Error correlation coefficient matrix	A matrix	Identity matrix (diagonal).
Uncertainty	PDF shape	Functional form	Gaussian
	units	Units	Counts
	magnitude		Given once per orbit file
Sensitivity coefficient		Value, equation or parameterisation of sensitivity of measurand to term	

Covariance Matrices – How they Simplify your Life!

$$u_c^2(y) = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j} u(x_i, x_j)$$

Simplifies to,

$$u_c^2(y) = \boldsymbol{C}_y \boldsymbol{V}_x \boldsymbol{C}_y^{\mathrm{T}}$$

For a single output quantity

Covariance Matrices – How they Simplify your Life!

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- C_y - Sensitivity coefficients

For a single output quantity

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Covariance Matrices – How they Simplify your Life!

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Simplifies to,

$$u_c^2(y) = C_y V_x C_y^{\mathrm{T}}$$

- C_y - Sensitivity coefficients
- V_x - Covariance Matrix

For a single output quantity

Covariance Matrices – Closer Look

$$u_c^2(y) = C_y V_x C_y^{\mathrm{T}}$$ (One output quantity)

- C_y - Sensitivity coefficients

$$C_y = \begin{pmatrix} \frac{\partial f}{\partial x_1} & \frac{\partial f}{\partial x_2} & \cdots & \frac{\partial f}{\partial x_n} \end{pmatrix}^{\mathrm{T}}$$

- V_x - Covariance Matrix

For a single output quantity

$$V_x = \begin{pmatrix} u^2(x_1) & u(x_1, x_2) & \cdots & u(x_1, x_n) \\ u(x_2, x_1) & u^2(x_2) & \cdots & u(x_2, x_n) \\ \vdots & \vdots & \ddots & \vdots \\ u(x_n, x_1) & u(x_n, x_2) & \cdots & u^2(x_n) \end{pmatrix}$$

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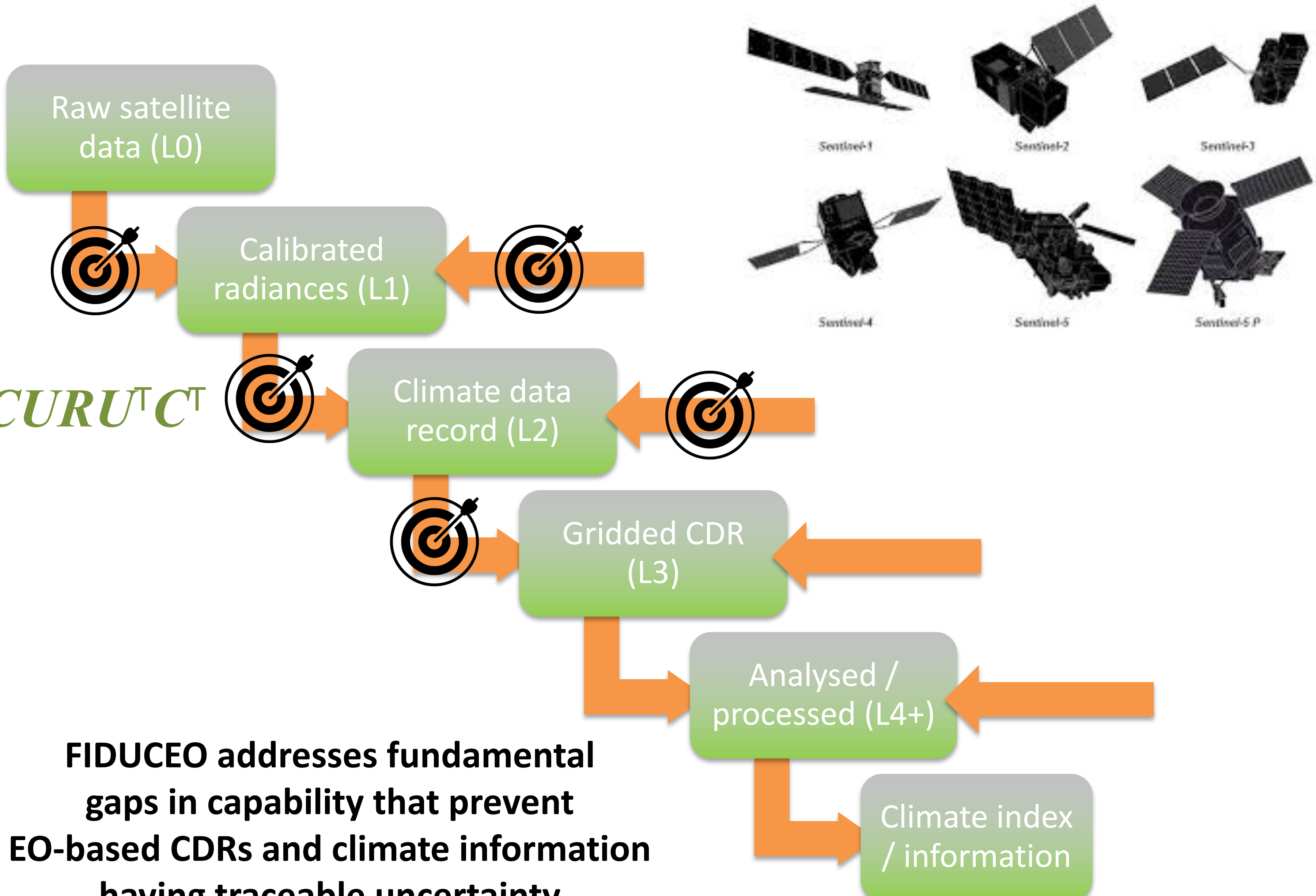
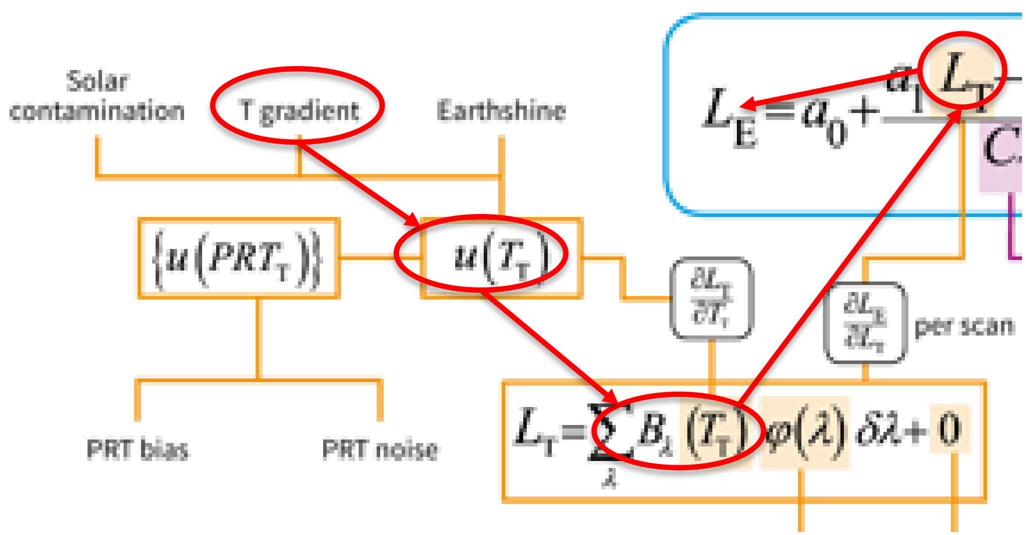


FIDUCEO addresses fundamental gaps in capability that prevent EO-based CDRs and climate information having traceable uncertainty

Source uncertainties - example

- Calibration Target temperature
  - Structured uncertainty component
  - Arises from errors in mapping estimates of calibration target temperature to calibration target radiances
  - Propagates through Planck function/spectral response function convolution to an uncertainty on the ICT radiance to a Gain uncertainty to the final Earth radiance
  - Uses the sensitivity coefficients to propagate uncertainties to the measurand



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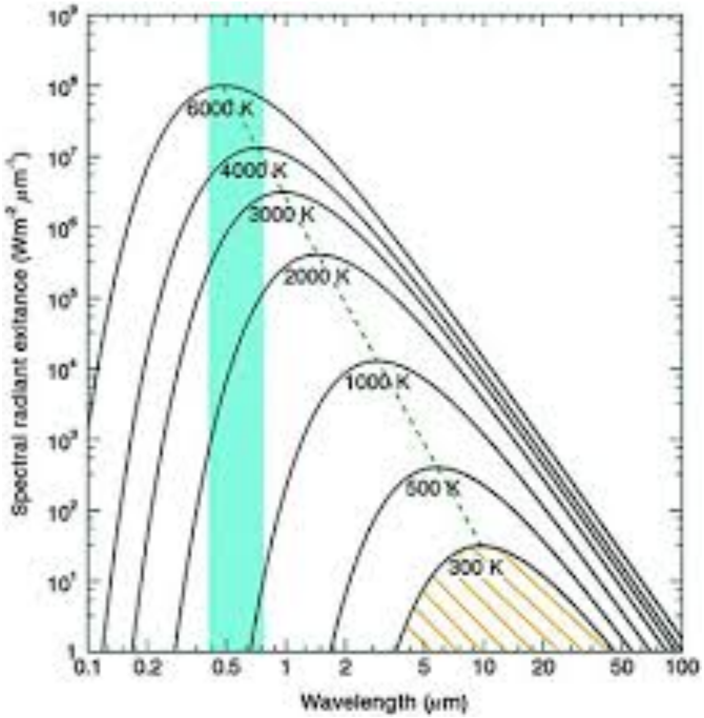
Introducing $\boldsymbol{CURU^{\mathrm{T}}C^{\mathrm{T}}}$

What is the covariance between the Earth radiance values in different spectral channels due to the uncertainty associated with the common effect in the internal calibration target temperature?

Introducing $CURU^{T}C^{T}$

$$\tilde{L}_{\mathrm{ICWT,A}} = \frac{\varepsilon_{\mathrm{A}} c_{1,L}}{\lambda_{\mathrm{A}}^{5}\left(\mathbf{exp}\left[c_2/\lambda_{\mathrm{A}}T\right]-1\right)}$$

$$\tilde{L}_{\mathrm{ICWT,B}} = \frac{\varepsilon_{\mathrm{B}} c_{1,L}}{\lambda_{\mathrm{B}}^{5}\left(\mathbf{exp}\left[c_2/\lambda_{\mathrm{B}}T\right]-1\right)}$$



An error in the temperature of the internal calibration target will affect all channels. But not equally

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Introducing $CURU^{\mathrm{T}}C^{\mathrm{T}}$

$$V_{\mathrm{LE},T} = \begin{pmatrix} c_{\mathrm{LA},T} & 0 & 0 \\ 0 & c_{\mathrm{LB},T} & 0 \\ 0 & 0 & c_{\mathrm{LC},T} \end{pmatrix} \begin{pmatrix} u_T & 0 & 0 \\ 0 & u_T & 0 \\ 0 & 0 & u_T \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} u_T & 0 & 0 \\ 0 & u_T & 0 \\ 0 & 0 & u_T \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} c_{\mathrm{LA},T} & 0 & 0 \\ 0 & c_{\mathrm{LB},T} & 0 \\ 0 & 0 & c_{\mathrm{LC},T} \end{pmatrix}^{\mathrm{T}}$$

Covariance matrix for Earth Radiances in different channels due to common temperature error

$u(T)$

Temperature uncertainty in K
The same throughout by definition

Full correlation

$$c_{\mathrm{LA},T} = \frac{\partial L_{\mathrm{E,A}}}{\partial L_{\mathrm{ICT,A}}} \frac{\partial L_{\mathrm{ICT,A}}}{\partial T}$$

Sensitivity coefficient to convert from temperature to Earth radiance uncertainty

More $CURU^{T}C^{T}$

Covariance matrix for Earth Radiances in different channels due to errors in Earth Counts

$$V_{LE,CE} = \begin{pmatrix} c_{LA,CE} & 0 & 0 \\ 0 & c_{LA,CE} & 0 \\ 0 & 0 & c_{LA,CE} \end{pmatrix} \begin{pmatrix} u_{CA} & 0 & 0 \\ 0 & u_{CB} & 0 \\ 0 & 0 & u_{CC} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_{CA} & 0 & 0 \\ 0 & u_{CB} & 0 \\ 0 & 0 & u_{CC} \end{pmatrix}^{T} \begin{pmatrix} c_{LA,CE} & 0 & 0 \\ 0 & c_{LA,CE} & 0 \\ 0 & 0 & c_{LA,CE} \end{pmatrix}^{T}$$

$u(C_{EA})$

Earth Count uncertainty Likely to change from channel to channel

No correlation

$$c_{LA,T} = \frac{\partial L_{E,A}}{\partial C_{E,A}}$$

Sensitivity coefficient to convert from Earth counts to Earth radiance uncertainty

Bringing $\boldsymbol{CURU}^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}}$ together

What is the covariance between the Earth radiance values in different spectral channels due to the uncertainty associated with the common effect in the internal calibration target temperature and the uncertainty associated with the independent effect Earth Counts?

$$\boldsymbol{V}_{L\mathrm{E}} = \sum_{\mathrm{Effects},\, i} \boldsymbol{C}_i \boldsymbol{U}_i \boldsymbol{R}_i \boldsymbol{U}_i^{\mathrm{T}} \boldsymbol{C}_i^{\mathrm{T}}$$

$$\boldsymbol{V}_{L\mathrm{E}} = \boldsymbol{C}_{C_{\mathrm{E}}} \boldsymbol{U}_{C_{\mathrm{E}}} \boldsymbol{R}_{C_{\mathrm{E}}} \boldsymbol{U}_{C_{\mathrm{E}}}^{\mathrm{T}} \boldsymbol{C}_{C_{\mathrm{E}}}^{\mathrm{T}} + \boldsymbol{C}_T \boldsymbol{U}_T \boldsymbol{R}_T \boldsymbol{U}_T^{\mathrm{T}} \boldsymbol{C}_T^{\mathrm{T}}$$

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Covariance Matrices SRF example

Example: Propagating model error covariance onto a curve

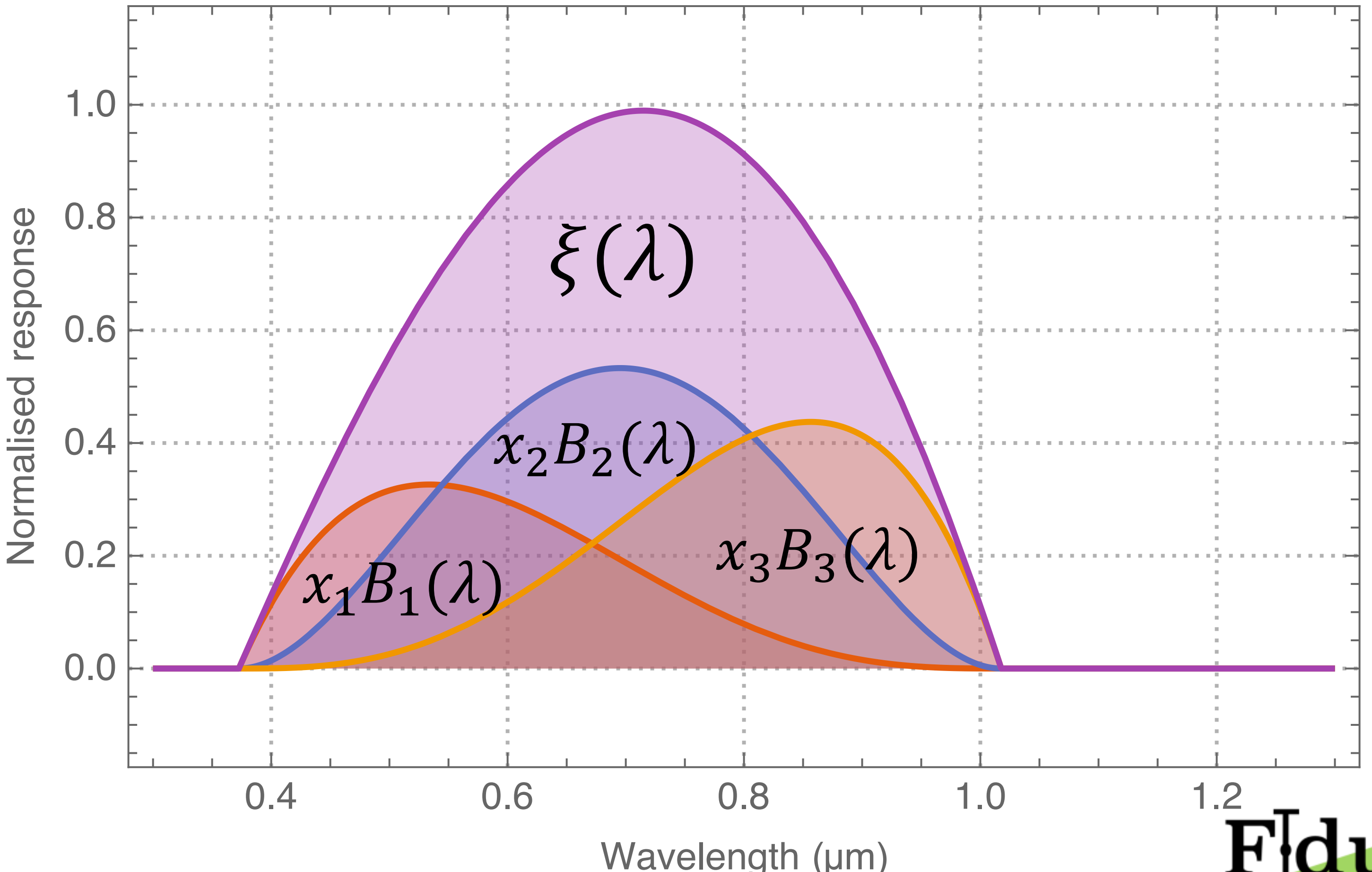
Spectral response model of the Meteosat visible broadband channel

$$\xi(\lambda) = f(\lambda, x_1, \dots, x_n)$$

Response model parameters $x_1, \dots, x_n$ have error covariance matrix V_x

Covariance Matrices Example III

Example: Propagating model error covariance onto a curve



Covariance Matrices Example III

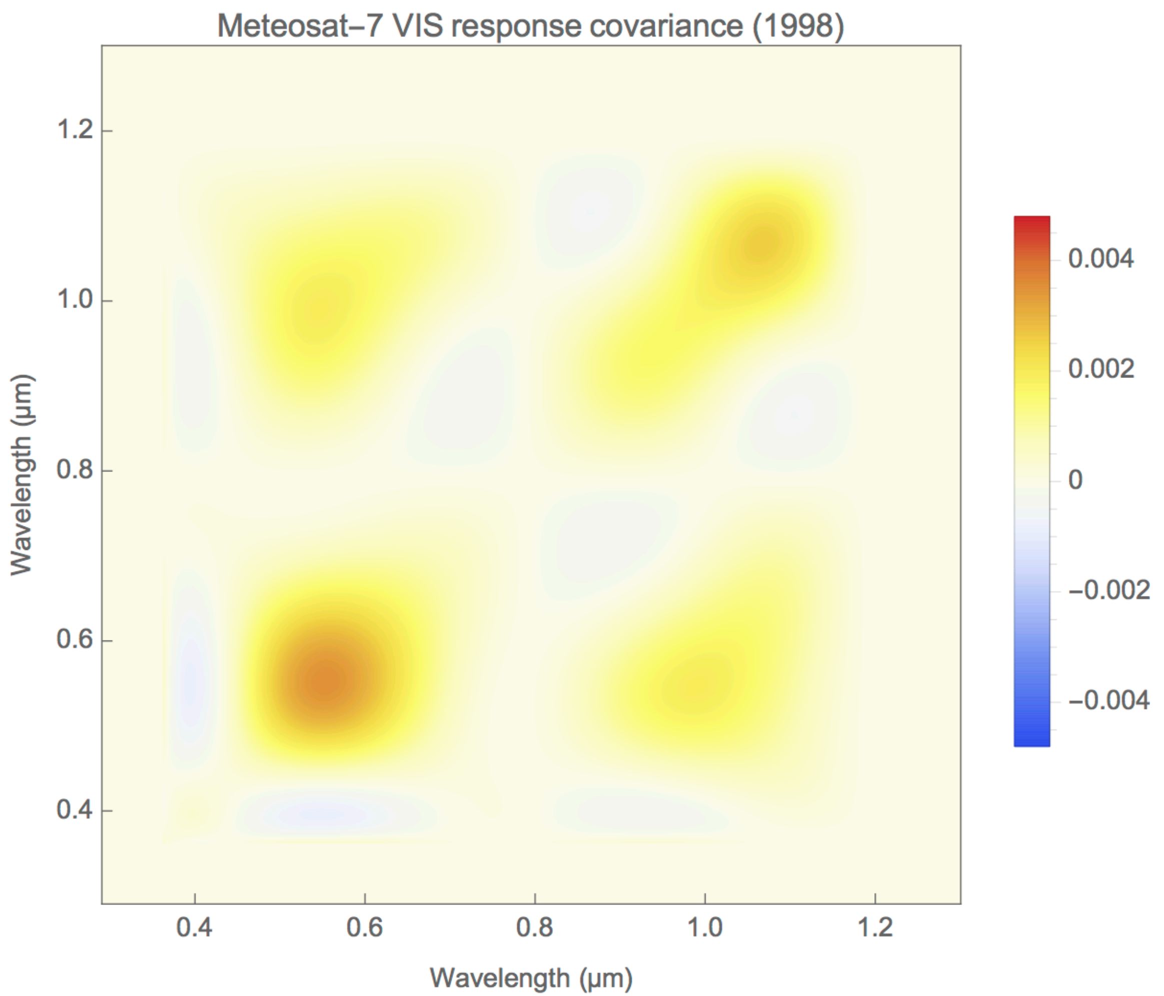
Example: Propagating model error covariance

What is the error covariance of the curve $\xi(\lambda)$?

$$V_{\xi}(\lambda) = C_f(\lambda) \times V_x \times C_f^{\mathrm{T}}(\lambda)$$

Covariance Matrices Example III

Example: Propagating model error covariance



Summary

- Error correlation structures (spatial and spectral) at the FCDR need to be understood to propagate to CDRs
- Propagation can be done with matrices using $\boldsymbol{CURU}^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}}$

FCDR Full & Easy

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Description of Scientific Content of FIDUCEO FCDRs

Chris Merchant (inputs from all FIDUCEO team during science meetings)
University of Reading

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