Introduction

"A good notation has a subtlety and suggestiveness which at times make it seem almost like a live teacher".

Bertrand Russell

This little booklet has been produced in response to a long standing problem, namely, that each lecturer in our Department tends to make use of his or her own favourite notation. The result is that links between courses can become obscured, and in a few unfortunate cases, students become totally confused.Starting in the autumn of 1991, we gathered together the various notations considered essential by each lecturer, and over the course of that term, negotiated an acceptable standard notation set. This has not been without pain; some cherished idiosyncrasies have been overturned, and in some cases, extensive editing of printed lecture notes will be needed. Our guidelines have been to seek consensus wherever possible. Where no consensus can be agreed, we have tried to follow the usage of the more commonly used textbooks. In one or two extreme cases, we have had to resort to quoting two notations for the same quantity to be used according to which course is being taken. This seems most frequent where quantities are used both in micrometeorology and synoptic meteorology. We quickly realised that it is impossible to be complete. Some of the more mathematically based subjects can use a vast number of different variables, some only for the purpose of one particular derivation. There are simply not enough letters in the Greek and Roman alphabets to give each variable a unique name. Lecturers will invariably invent a notation on the spur of the moment to make a particular point. What we have tried to do is to identify those variables that are likely to be used by more than one course. These need to be as consistent as possible. It is our hope that over a period of time, these suggested notations will become normal practice in the Department, both in teaching and in writing essays and theses, and even in preparing research papers. Poor and inconsistent notation is a barrier to communication and makes for inefficient learning. On the other hand, as the quotation from Russell suggests, the benefits of a good notation go beyond mere administrative tidiness, and bring an intellectual and aesthetic vigour to our science. Please tell us if you find any errors or omissions in this booklet, or if you have ideas for better notations at any point. Above all, please use the booklet!

Ian N. James,
2/9/1992.

Introduction to the 1998 revision

Staff change, courses change and students change! The need for a reprinted Notation Booklet has given us the opportunity to revise it, adding some new sections and revising others. Time has convinced me even more strongly of the need for this booklet, and of the impossibility of pleasing everyone with its contents.

Ian N. James
7/10/1998.

Introduction to the 2001 revision

While flying over the Dolomite mountains, Prof. Brian Hoskins happened to mention that he used a standard notation in his lectures that was described in a small departmental booklet. I tracked down a surviving copy of this booklet and found it to be an invaluable guide. To bring its benefits to more people, I have produced this revised version both in hard copy booklet format and in an online format available via the Department of Meteorology web pages (www.met.rdg.ac.uk). Please do not hesitate to send me any suggestions for future revisions.

David B. Stephenson
25/11/2001.

1. Mathematical notations

exp(x)	$\equiv e^x$, convenient when x is a long expression.
ln	Natural logarithm, base e.
log	Logarithm to base 10.
$\frac{d}{dx}$	Ordinary derivative of a function of x alone.
$\frac{\partial}{\partial x}$	Partial derivative of a function with respect to x holding all other variables constant
$\frac{\partial}{\partial t}$	Eulerian time derivative.
$\frac{D}{Dt}$	Lagrangian derivative.
Q_x	$\equiv \partial Q/\partial x$, a convenient shorthand.
$\mathbf{a}$	Vector.
$\mathbf{a.b}$	Scalar product.
$\mathbf{a}\times\mathbf{b}$	Vector product. (sometimes denoted $\mathbf{a}\wedge\mathbf{b}$)
δx	Small increment of x
Δx	Finite increment of x.
∇	Vector gradient operator (variously pronounced “del”, “grad” or “nabla”.

2. Units

Systéme Internationale (SI) units should be used whenever possible.

Length: metre (m)	Mass: kilogram (kg)	Time: second (s)
Force: Newton (N)	Energy: Joule (J)	Power: Watt (W)
Temperature: Kelvin (K)	Pressure: Pascal (Pa)	

Powers of 10: SI prefers units to be renamed in steps of 10^3. The steps are:

10^3 - kilo (k)	10^6 - mega (M)	10^9 - giga (G)	10^{12} - tera (T)
10^{-3} - milli (m)	10^{-6} - micro (μ)	10^{-9} - nano (n)	10^{-12} - pico (p)

Certain secondary and unconventional units are traditional in meteorology, including many non-SI units in operational meteorology. Among the non-standard units in frequent use are:

Temperature: °C, pressure: millibars (mb, also sometimes called the "hectoPascal", hPa), time: days (1 sidereal day is 86170 s), mass: tonne (t).

In an operational context, horizontal wind speeds are often given in knots (nautical miles per hour) where the nautical mile is the distance subtended by 1 minute of latitude at the Earth's surface. Heights are given in feet for operational purposes (a consequence of the frequent application of meteorology to aviation).

$$1 \text{ knot} = 0.514 \text{ m s}^{-1}, \ 1 \text{ foot} = 0.3048 \text{ m}$$

3. Averaging operators

For any meteorological variable, Q:

$[Q]$	Zonal mean
Q^*	Departure from zonal mean
$\overline{Q}$	Time mean over some averaging period t; t may be a season (global circulation) or a few minutes or seconds (turbulence).
Q'	Departure from time mean
$\hat{Q}$	Mass weighted vertical mean
$\langle Q \rangle$	Global mean (often normalised so it may be expressed in units of Q m^{-2})
$\tilde{Q}$	Ensemble average (for example, the average of several seasons to form a climatology).

4. Physical properties of the Earth

Symbol	*Meaning*	*Value*
a	Mean radius of Earth	6.371×10^{6} m
g^*	Local acceleration due to gravity	
g	Global mean acceleration due to gravity	9.81 m s^{-2}
Ω	Rotation rate of Earth	7.292×10^{-5} s^{-1}

5. Coordinate systems

Symbol	*Meaning*
r	Distance from centre of Earth
r (or $\vec{r}$)	Position vector
u	Eastward (zonal) component of wind
u (or $\vec{u}$)	Velocity vector $= u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$
v	Northward (meridional) component of wind
v (or $\vec{v}$)	Horizontal component of velocity vector $= u\mathbf{i} + v\mathbf{j} + 0\mathbf{k}$
w	Vertical component of velocity
x	Distance in eastward direction
y	Distance in northward direction
z	Geometric height above mean sea level
Z	Geopotential height above mean sea level
z'	"Pseudo-height" $= -H \ln(p / p_R)$ where H is a constant scale height.
ϕ	Latitude
λ	Longitude

6. Properties of dry air

Symbol	*Meaning*	*Value or units*
c_p	Specific heat at constant pressure	1004 J K^{-1} kg^{-1}
c_v	Specific heat at constant volume	717 J K^{-1} kg^{-1}
M or M_d	Mean molecular weight of dry air	29
p	Pressure	Pa
p_s	Surface pressure (not adjusted to mean sea level).	Pa
p_R	Standard reference pressure	100 kPa
p_0	Surface pressure adjusted to mean sea level	Pa
R or R_d	Gas constant for dry air	287 J K^{-1} kg^{-1}
R^*	Universal gas constant	8314 J K^{-1} (kg mole)$^{-1}$
T	Temperature	K
α	Specific volume $\equiv 1/\rho$	m^3 kg^{-1}
γ	Ratio of specific heats c_p/c_v	1.400
Γ	Lapse rate $\equiv -\partial T/\partial z$	K m^{-1}
Γ_d	Dry adiabatic lapse rate	9.77 K km^{-1}
κ	R_d/c_p	0.286
θ	Potential temperature	K
ρ	Density	kg m^{-3}
ν	Kinematic viscosity of air	1.35×10^{-5} m^2 s^{-1}

7. Properties of moist air and of water

Symbol	*Meaning*	*Value or units*
c_{pv}	Specific heat of water vapour at constant pressure.	1925 J $K^{-1}kg^{-1}$
c_L	Specific heat of liquid water	4218 J $K^{-1}kg^{-1}$
e	Vapour pressure of water	Pa
e_s	Saturated vapour pressure of water	611 Pa at 273 K
L (or λ)	Latent heat of condensation of water	2.50×10^6J kg^{-1} at 273 K
M_v	Molecular weight of water	18
q	Specific humidity	Dimensionless
r	Humidity mixing ratio	Dimensionless
R_v	Gas constant for water vapour	461.5 J $K^{-1}kg^{-1}$

T_d	Dewpoint temperature	K
T_v	Virtual temperature	K
T_w	Wet bulb temperature	K
ε	$= R_d / R_v \equiv M_v/M_d$	0.622
ρ_L	Density of liquid water	10^3 kg m^{-3} at 273 K
θ_E	Equivalent potential temperature	K
θ_w	Wet bulb potential temperature	K
ρ_V (or χ)	Absolute humidity	kg m^{-3}

8. **Atmospheric radiation**

Symbol	*Meaning*	*Value or units*
a	Absorptance	/
B	Planck function radiance	W m^{-2} sr^{-1}
E	Irradiance (sometimes flux density *F)*	W m^{-2}
k_λ	Monochromatic extinction coefficient	m^{2}kg^{-1}
L	Radiance (sometimes intensity *I*)	W m^{-2} sr^{-1}
M	Exitance	W m^{-2}
P	Phase function	/
s	Path length	m
S_0	The “solar constant”	1370 W m^{-2}
T_E	Radiative equilibrium temperature	K
$\tilde{w}$	Single scattering albedo	/
α	Reflectance (albedo)	/
α_p	Planetary albedo	/
α_s	Surface albedo	/
δ	Optical depth (sometimes denoted τ)	/
ε	Emittance	/
θ	Zenith angle	rad
μ	Cosine of solar zenith angle	/
τ	Transmittance (sometimes denoted T_r)	/
τ_E	Radiative equilibrium timescale	s
σ	Stefan constant	5.67×10^{-8}W m^{-2}K^{-4}
Ω	Solid angle	sr

9. **Surface energy balance**

NB - the sign convention is that all radiation fluxes are positive TOWARDS the surface; other fluxes are positive AWAY from the surface.

Symbol	*Meaning*	*Value or units*
E	Evaporation rate	kg m^{-2} s^{-1}
G_0	Ground heat flux density at surface	W m^{-2}
G	Ground heat flux density	W m^{-2}
H	Sensible heat flux density	W m^{-2}
L_u	Upward longwave irradiance	W m^{-2}
L_d	Downward longwave irradiance	W m^{-2}
L_n	Net longwave irradiance	W m^{-2}
P	Precipitation rate	kg m^{-2}s^{-1}
R_n	Net all wave irradiance	W m^{-2}
S	Shortwave irradiance on a horizontal surface	W m^{-2}
S_d	Diffuse shortwave irradiance	W m^{-2}
S_b	Direct beam irradiance at normal incidence	W m^{-2}
S_g	Global solar irradiance	

10. **Boundary layer meteorology**

Symbol	*Meaning*	*Units*
c_D	Drag coefficient	/
d	Displacement height for momentum	m
k	Thermal conductivity	J m^{-1}s^{-1} K^{-1}
k	Von Karman’s constant	/
K	Eddy diffusion coefficient	m^{2} s^{-1}
K_M	Eddy diffusion coefficient for momentum	m^{2} s^{-1}
K_T	Eddy diffusion coefficient for temperature	m^{2} s^{-1}
K_v	Eddy diffusion coefficient for water vapour	m^{2} s^{-1}
L	Obukhov length	m
u_*	Friction speed	m s^{-1}
z_0	Roughness length	m
τ_0	Surface stress	N m^{-2}
τ_D	Drag (or “spin up”) timescale	s

11. **Dynamical meteorology**

Symbol	*Meaning*	*Value or units*
c	Phase speed	m s^{-1}
$\mathbf{c}_g \equiv \left(c_{gx}, c_{gy}, c_{gz}\right)$	Vector group velocity	m s^{-1}
D	"Horiozontal divergence", $u_x + v_y$	s^{-1}
$f \equiv 2\Omega \sin\phi$	Coriolis parameter	s^{-1}
H	Pressure scale height	m
$h(p) \equiv \frac{R}{p}\left(\frac{p}{p_R}\right)^{\kappa}$	A useful function of pressure.	m K^{-1}kg^{-1}
$\mathbf{k} \equiv (k, l, m)$	Vector wavenumber	m^{-1}
$K \equiv \lvert\mathbf{k}\rvert$	Total wavenumber	m^{-1}
M	Montgomery potential	m^{2}s^{-2}
N	Brunt-Väisälä frequency	s^{-1}
P	Ertel potential vorticity	K m^{2}s^{-1}kg^{-1}
q	Quasi-geostrophic potential vorticity	s^{-1}
$\dot{Q}$	Heating rate	W kg^{-1}
$\hat{R}(p)$	Alternative notation for *h(p)*	m K^{-1}kg^{-1}
$\mathbf{u} \equiv (u, v, w)$	Flow velocity	m s^{-1}
$\mathbf{v} \equiv (u, v, 0)$	Horizontal component of wind	m s^{-1}
$\mathbf{v}_g \equiv \left(u_g, v_g, 0\right)$	Geostrophic wind	m s^{-1}
$\mathbf{v}_a \equiv \left(u_a, v_a, 0\right)$	Ageostrophic part of horizontal wind	m s^{-1}
Z	Geopotential height	m
β	$\equiv \partial f / \partial y$	m^{-1}s^{-1}
Φ	The geopotential $\equiv gZ$	m^{2}s^{-2}
ζ (or $\vec{\zeta}$)=$(\zeta_1, \zeta_2, \zeta_3)$	Vector absolute vorticity	s^{-1}
ζ	Vertical component of absolute vorticity	s^{-1}
ξ (or $\vec{\xi}$)=(ξ_1, ξ_2, ξ_3)	Vector relative vorticity	s^{-1}
ξ	$\equiv \zeta - f$, vertical component of relative vorticity	s^{-1}
ψ	Streamfunction	m^{2}s^{-1}
ψ_g	Geostrophic streamfunction	m^{2}s^{-1}

Π	Exner function	$m^2s^{-2}K^{-1}$
σ	Growth rate of unstable or damped wave	s^{-1}
ω	Pressure vertical velocity $\equiv Dp / Dt$	Pa s^{-1}
ω	Also used for the frequency of a wave	s^{-1}

12. **Atmospheric chemistry**

It is usual to use cgs rather than S.I. units in chemistry.

Symbol	*Meaning*	*Units*
$[CH_4]$	Concentration of (e.g.) methane	molecules cm^{-3}
D.U.	Dobson unit	2.688×10^{16} molecules cm^{-2}
F	Active monochromatic flux	photons $cm^{-2}s^{-1}nm^{-1}$
k	Rate constant - bimolecular	$cm^3molecule^{-1}s^{-1}$
	- trimolecular	$cm^6molecule^{-2}s^{-1}$
J	Photolysis frequency	s^{-1}
N_{LOS}	Loschmidt's number	2.688×10^{19} molecules cm^{-3}
σ_λ	Absorption cross section	cm^2 $molecule^{-1}$
τ_{CH4}	The e-folding lifetime of (e.g.) CH_4	s
ϕ_λ	Quantum yield	Dimensionless
χ_{CH4}	Mixing ratio of (e.g.) methane by volume	Dimensionless

13. **Commonly used dimensionless numbers**

Number	*Name*	*Definition*
Bo	Bowen ratio	$H/(\lambda E)$
Bu	Burger	$N^2H^2/(f^2L^2)$
Fr	Froude	$U/(gL)^{1/2}$ (external)
		$U/(Nh)$ (internal)
Ek	Ekman	$\nu/(2\Omega D^2)$
Kn	Knudsen	λ / a
Nu	Nusselt	$HL/(\kappa\Delta T)$
Pr	Prandtl	ν/κ
Ra	Rayleigh	$N^2h^4/(\nu\kappa)$
Re	Reynolds	UL/ν
Ri	Richardson	$N^2/(\partial u/\partial z)^2$
Ro	Rossby	$U/(\Omega L)$

14. **Notation used in statistics**

- Upper case roman letters are used to denote random variables (e.g. X, Y, Z, etc.), whereas lower case roman letters are used to denote their specific values (e.g. x,y,z, etc.); for example, the probability of random variable X exceeding a specific value x is given by p(X>x). When referring to sample variables use lower case roman letters.
- Not directly observable quantities such as model parameters and noise are denoted using lower case Greek letters e.g. α, β, γ etc.; for example, $Y = X\beta + \mu + \varepsilon$ for the linear regression model.
- The hat symbol is used to denote estimated and predicted values; for example, $\hat{\beta}$ is an estimate of the parameter β, and $\hat{Y}$ is an estimate or prediction of the random variable Y.
- Bold face upper case roman letters are used to denote data matrices containing multiple variables. For example, the (nxp) data matrix **X** contains elements x_{ij} where index $i = 1,2,\ldots,n$ indicates the sample time and index $j = 1,2,\ldots,p$ indicates the variable.
- The symbol, n, is used to denote the sample size i.e. the number of objects in the sample.
- The population mean is denoted by the "expectation" operator e.g. $E(x)$, whereas the sample mean is denoted by using overbars e.g. $\bar{x}$ for sample mean of variable x.

15. **The Greek alphabet**

Lower case	*Anglicized pronunciation*	*Upper case*
α	alpha	Α
β	beta	Β
γ	gamma	Γ
δ	delta	Δ
ε	epsilon	Ε
ζ	zeta	Ζ
η	eta	Η
θ	theta	Θ
ι	iota	Ι
κ	kappa	Κ
λ	lamda	Λ
μ	mu	Μ
ν	nu	Ν
ξ	xi	Ξ
ο	omicron	Ο
π	pi	Π
ρ	rho	Ρ
σ	sigma	Σ
τ	tau	Τ
υ	upsilon	Υ
φ	phi	Φ
χ	chi	Χ
ψ	psi	Ψ
ϕ	alternative psi	
ω	omega	Ω

Notes